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Mašhad, Kitābḫāna-i Āsitān-i Quds-i Raḍawī 300, f. 1v
Paris, Bibliothèque nationale de France, *grec* 1853, f. 186v

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# *Al-Kindī, Ptolemy (and Nicomachus of Gerasa) Revisited*

Emma Gannagé*

To Gerhard Endress and Dimitri Gutas
*We ought to be ... even more grateful to*
*those who have contributed much of the truth.*
al-Kindī, *On First Philosophy*

*Abstract*
Al-Kindī's classification of mathematics as an intermediary science between theology, on the one hand, and physics on the other, that is reiterated in several of his treatises, has been so far generally traced back to Proclus. Based on the introduction of al-Kindī's treatise *On the Great Art* (*Fī l-Ṣināʿa l-ʿuẓmā*), as well as on other works of his, this article shows the rather structural influence of Ptolemy's philosophy on al-Kindī. From Ptolemy al-Kindī draws not only the division of theoretical sciences, but also a philosophical program that gives mathematics a central position as a full theoretical science. This paper tries to reconcile between the thesis of the intermediary position of mathematics held in these treatises and its role as a propaedeutic to the study of philosophy in al-Kindī's *Epistle On the Quantity of Aristotle's Books and What is Necessary for the Attainment of Philosophy*. It shows that the quadripartition of mathematics al-Kindī laid out in this epistle is drawn from an 'altered' version of Nicomachus of Gerasa's *Introduction to Arithmetic*. The reading he found in that version seems to have bolstered al-Kindī's argument aimed at promoting the science of quality and quantity, namely mathematics, as the leading path towards the knowledge of the first and secondary substances, hence elevating this science as the best guide to reality. Such description mirrors the conception of mathematics provided by Ptolemy in the *Almagest*.

Al-Kindī's treatise *On the Great Art* (*Kitāb Fī l-Ṣināʿa l-ʿuẓmā*) was brought to light by the great scholar Franz Rosenthal, in a seminal article whose importance has not yet received all the attention it deserves.[1] The treatise is a paraphrase of the first 8 chapters of the first book of Ptolemy's *Almagest*.[2]

* Different versions of this paper have been read at: the Catholic University of America, School of Philosophy 2013 Fall Lecture Series, "Philosophy in the Islamic Lands"; the Society for Medieval and Renaissance Philosophy Session at the American Philosophical Association annual meeting, Baltimore, Dec. 2013; The Institute for Advanced Study, Princeton, Nov. 2013; The Islamic-Arabic Philosophy Workshop, Harvard University, Oct. 2014 and The Center for Middle Eastern Studies, University of California Berkeley, Nov. 2014. I am particularly grateful to Thérèse-Anne Druart, Jon McGinnis, Julie Klein, Maria Mavroudi, Khaled El-Rouayheb, Jeffrey McDonough, and to the late Patricia Crone for their kind invitations and to the different audiences for stimulating questions and comments. I would also like to thank the anonymous reader of this article for his helpful remarks.

[1] See F. Rosenthal, "Al-Kindī and Ptolemy", in *Studi orientalistici in onore di Giorgio Levi della Vida*, 2 vols, Istituto per l'Oriente, Roma 1956 (Pubblicazioni dell'Istituto per l'Oriente, 52), vol. 2, pp. 436-56. The treatise came down to us in a unique manuscript, Istanbul, Aya Sofya 4830, ff. 53r-80v. See also Yaʿqūb b. Isḥāq al-Kindī, *Fī l-Ṣināʿa al-ʿuẓmā*, ed. ʿAzmī Ṭāha al-Sayyid Aḥmad, Dār al-Šabāb, Cyprus 1987. My warmest thanks to Prof. Maroun Aouad, director of the ERC Project "Philosophy in Context: Arabic and Syriac Manuscripts in the Mediterranean World (PhiC)," for having kindly provided me access to the manuscript.

[2] The Greek title of the *Almagest* is Μαθηματικὴ Σύνταξις, which means "mathematical [that is, astronomical]

Rosenthal already noticed that the treatise ends with the mention of a sequel ("let us complete this chapter of our book and follow it up with what naturally comes after, *fa-li-nukmil hāḏa l-fann min kitābinā wa-l-nutlihi bi-mā yatlū ḏālika tilwan ṭabīʿiyyan*") that seems to indicate that the text which we have now is incomplete.[3] This being said, the six chapters of *Almagest* Bk 1 that come immediately after chap. 2, which lays out the general outline of the book, have been singled out by Ptolemy himself as a consistent preliminary unit dealing with "general" considerations by way of "an introduction to the discussion of particular topics and what follows after".[4] Indeed, they include a brief description of the spherical motion of the heavens (Ch. 3 and 8), the shape, position, size and immobility of the earth (Ch. 4-7), in sum a concise picture of the universe considered as prerequisites for the understanding of the planetary motion theory that will follow.[5] Therefore, whether as a result of an accident of transmission or of a deliberate selection, the treatise, as it stands, forms a consistent unit, as already noticed by Rosenthal (p. 437).

As it has reached us, in the Aya Sofya manuscript, the beginning of *Fī l-Ṣināʿa l-ʿuẓmā* is slightly perplexing. It bears a multi-layered prologue, which opens with a first *incipit* written like a bibliographic record. These first lines mention the title of the treatise, the identity of the dedicatee, namely the author's own son Aḥmad b. Yaʿqūb, and a table of contents of the 8 chapters at stake that reproduces partially the one found at the beginning of al-Ḥaǧǧāǧ ibn Maṭar's translation of the *Almagest*.[6] Obviously this *incipit* must have been appended to the original treatise at some point during the transmission process. It is immediately followed by the treatise properly said, that starts afresh with a new *basmala* followed by a direct address to the dedicatee of the work, who remains anonymous. We nevertheless learn that he has requested a "description" (*rasm*)[7] of the

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compilation". According to G.J. Toomer, "In later antiquity it came to be known informally as *ê megalê suntaxis* or *ê megistê suntaxis* (the great [or greatest] compilation), perhaps in contrast with a collection of earlier Greek works on elementary astronomy called *o micros astronomoumenos* (the small astronomical collection). The translators into Arabic transformed *ê megistê* into 'al-majisti,' and this became 'almagesti' or 'almagestum' in the Medieval Latin translations" (art. "Ptolemy", *Dictionary of Scientific Biography*, Scribner, New York 1975, vol. XI, pp. 186-206 [p. 187]). For further discussion on the Arabic title see P. Kunitzsch, *Der Almagest: Die Syntaxis Mathematica des Claudius Ptolemäus in arabisch-lateinscher Überlieferung*, Harrassowitz, Wiesbaden 1974, pp. 115-25. The *Almagest* has been translated very early and more than once into Arabic. At least five different versions, if not more, have been identified. For a detailed discussion of the varied and complex Arabic transmission of the *Almagest* see *ibid.*, pp. 15-82. For the version that al-Kindī might have had at hand, see below, n. 15.

[3] See Rosenthal, "al-Kindī and Ptolemy" (quoted above, n. 1), pp. 437-8, who points to the striking similarity between this concluding remark and the one that closes al-Kindī's book *On First Philosophy* as it has reached us (see *Kitāb al-Kindī ilā l-Muʿtaṣim bi-llāh fī l-falsafa l-ūlā*, in *Rasāʾil al-Kindī al-falsafiyya*, ed. M. ʿA. Abū Rīda, Dār al-Fikr al-ʿArabī, Cairo 1950, vol. 1, p. 162 and R. Rashed - J. Jolivet, *Œuvres philosophiques et scientifiques d'al-Kindī*, *Vol. II: Métaphysique et Cosmologie*, Brill, Leiden 1998 [Islamic philosophy, theology, and science, 29], p. 99).

[4] See *Claudii Ptolemaei Opera quae exstant omnia, Vol. I: Syntaxis Mathematica, Pars I: libros VII - XIII continens*, ed. J.L. Heiberg, Teubner, Leipzig 1898, Bk. I 8, p. 26; Engl. tr. G.J. Toomer, *Ptolemy's Almagest*, Duckworth, London 1984 (Duckworth Classical, Medieval and Renaissance Editions), p. 45.

[5] See O. Pedersen, *A Survey of the Almagest*, with Annotation and New Commentary by A. Jones, Springer, New York 2011 (Sources and Studies in the History of Mathematics and Physical Sciences), pp. 35-46.

[6] This translation came down to us in two manuscripts: the complete Leiden, *Or.* 680 and the incomplete London, British Library, *Add.* 7474. According to the incipit of the ms. Leiden, *Or.* 680 (f. 2v 3) al-Ḥaǧǧāǧ ibn Yūsuf ibn Maṭar al-Ḥāsib (*fl.* 786-830) is said to have translated the *Almagest* with Sarǧūn ibn Hilyā al-Rūmī for the caliph al-Maʾmūn in 212h./827-28. On this version, see Kunitzsch, *Der Almagest* (quoted above, n. 2), pp. 64-7.

[7] For *rasm* see the title of *Risāla fī Ḥudūd al-ašyāʾ wa-rusūmihā* (*Epistle on the Definition and Description of Things*), and Al-Kindî, *Cinq Épîtres*, CNRS-Éditions, Paris 1976 (Centre d'Histoire des sciences et des doctrines. Histoire des sciences et de la philosophie arabes), p. 39. *Rasama* will also be used throughout the prologue in the sense of 'writing', 'designing' and the likes.

great art (here *al-ṣināʿa l-kubrā*)[8] due to the great opacity (*šiddat al-istiġlāq*) of the description provided by Ptolemy. The avowedly "didactic purpose"[9] of the book is thus stated clearly from the outset and reiterated a few lines below, namely "to expound [the great art] simply, making plain the roughness of its paths and shedding light on its procedures".[10] Hence al-Kindī claims to have written

> An Introduction (*madḫalan*) to this art, following the model we are imitating concerning its pattern and what [Ptolemy] had written about it (*rasm al-rāsim fīhā*), addressing its parts, its arrangement, its aim and what is proper to each one of its parts and what is concomitant to it, in a concise and plain manner that simplifies its perception (*al-musahhil wuǧūdahā*).[11]

The imitated model is thus clearly Ptolemy's Μαθηματικὴ Σύνταξις. However, al-Kindī is following here a pattern that he finds in Theon of Alexandria's commentary on the *Almagest*,[12] as Theon used to consider the source on which he was commenting not only as a "subject-matter" but also as a "model".[13] Al-Kindī was indeed reading the *Almagest* with the guidance of Theon's commentary from which he drew extensively, as already noticed by Rosenthal. Like his source, he opens his paraphrase with a prologue of his own,[14] but while Theon's own prologue is quite short, al-Kindī's one is much longer and borrows extensively from Theon's own commentary on the preface of the *Almagest*. Actually, it follows closely Ptolemy's preface, mainly through the lens of Theon's commentary, but also expands on it.

Finally, al-Kindī's own introduction, although emulating closely Ptolemy's preface, as we have just said, is still followed by the actual and almost literal paraphrase of the latter. In itself it constitutes

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[8] For Rosenthal, "al-Kindī and Ptolemy" (quoted above, n. 1), p. 438, the title of the treatise is problematic as it would be sometimes referred to as *al-Ṣināʿa l-ʿuẓmā* and sometimes as *al-ṣināʿa l-kubrā*. However, *al-Ṣināʿa l-ʿuẓmā* seems to be the title given systematically by al-Kindī, in this text, to Ptolemy's *Almagest* (we find it mentioned twice in the core of the text, see p. 148.18 and p. 200.1 Aḥmad). Still, he also designates often Ptolemy's treatise by its transliterated title *al-Maǧisṭī*, that he describes as dealing with *al-ṣināʿa l-kubrā*. The latter denomination seems thus to refer to the discipline of astronomy.

[9] I borrow the expression from A. Bernard, "In What Sense did Theon's commentary on the *Almagest* have a Didactic Purpose?", in A. Bernard - C. Proust (eds.), *Scientific Sources and Teaching Contexts Throughout History: Problems and Perspectives*, Springer, Dordrecht 2014 (Boston Studies in the Philosophy and History of Science, 301), pp. 95-121.

[10] Al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, p. 120.3 Aḥmad.

[11] *Ibid.*, p. 120.15-17 (amended).

[12] See Rosenthal, "al-Kindī and Ptolemy" (quoted above, n. 1), pp. 446 ff., who shows the close dependence of al-Kindī's treatise on Theon's *Commentary* on the *Almagest*. According to A. Jones, already by the 4th century Ptolemy's *Almagest* was read along with Pappus's and Theon's commentaries (see A. Jones, "Uses and Users of Astronomical Commentaries in Antiquity", in G.W. Most [ed.], *Commentaries – Kommentare. Aporemata: Kritische Studien zur Philologiegeschichte*, Band 4, Vandenhoeck & Ruprecht, Göttingen 1999, pp. 147-72 [p. 168], quoted by Bernard, "In What Sense", p. 112). Ibn al-Nadīm's *Fihrist* mentions under the entry on Theon of Alexandria an "*Introduction* to the *Almagest* in an old translation" (*Kitāb al-madḫal ilā l-maǧisṭī bi-naqlin qadīm*), which is apparently lost. See Ibn al-Nadīm, *Kitāb al-Fihrist*, ed. G. Flügel -J. Rödiger - A. Müller, 2 vols., Vogel, Leipzig 1871-1872, vol. I (1871), p. 268.

[13] See Bernard, "In What Sense" (quoted above, n. 9), p. 99.

[14] It is worth noting that Theon's commentary is also addressed to his son Epiphanos, though as noted by A. Rome, the term τέκνον can be considered as an endearing term not necessarily meaning that Epiphanos was Theon's son, see *Commentaires de Pappus et de Théon d'Alexandrie sur l'Almageste*, Texte établi et annoté par A. Rome, t. II: *Théon d'Alexandrie, Commentaire sur les livres 1 et 2 de l'Almageste*, Biblioteca Apostolica Vaticana, Città del Vaticano 1936 (Studi e Testi, 72), p. 317 n. 3. Strikingly enought the dedicatee of al-Kindī's own prologue remains anonymous, whereas the *incipit* of the treatise identifies him as the philosopher's own son, Aḥmad b. Yaʿqūb.

the third layer of this prologue and is introduced as follows: "This is what Ptolemy has set forth in the book *Almagest*, and with which he introduced his book" (*wa-hāḏa mā qaddama Baṭlamyūs fī kitāb al-maǧisṭī wa-ṣādara bihi kitābahu*).[15]

Such a multilayered composition raises some concerns as to the nature of the text at stake: is al-Kindī's prologue to *On the Great Art* a mere paraphrase of his two famous predecessors, or does it draw the contours of an original project that finds a larger echo in al-Kindī's overall philosophy, admittedly inspired by Ptolemy and his commentator?

The question is all the more crucial, given that the *Almagest* itself is an avowedly philosophical treatise that opens with a bold statement endorsing the distinction drawn by "the legitimate philosophers" between "the theoretical part of philosophy" and the practical one.[16] Furthermore, it subordinates the latter to the former on which it is somewhat dependent. In the same breath, Ptolemy adopts Aristotle's division of theoretical philosophy in three categories, namely physics, mathematics and theology (*Metaph.* VI 1) and gives mathematics a central role in both respects: not only is mathematics the only one among the three theoretical sciences that provides sure and incontrovertible knowledge, but it is also the only one that is meant, through the cultivation of theoretical astronomy, to enable the conversion of one's nature to a spiritual state through "the constancy, order, symmetry and calm which are associated with the divine" as Ptolemy advocates in his preface.[17]

Following in the footsteps of Ptolemy, al-Kindī proceeds to distinguish between practical and theoretical philosophy, and divides the latter into three divisions: physics or natural science, mathematical science (*al-ʿilm al-taʿlīmī*),[18] and theology.[19] This division rests on the nature of the

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[15] See al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, p. 130.15ff Aḥmad. A quick comparison with the mss. Leiden, *Or*. 680 and London, British Library, *Add*. 7474 confirms that al-Kindī should have had at hand the translation of al-Ḥaǧǧāǧ ibn Yūsuf ibn Maṭar (786-833) and Sarǧūn ibn Hilyā al-Rūmī, that he reproduces almost literally with some few changes. I would like to thank particularly Dr. Bink Hallum of the British Library for having kindly provided me with access to the latter manuscript. See also Rosenthal, "al-Kindī and Ptolemy" (quoted above, n. 1), p. 439, n. 2.

[16] See Ptolemy, *Almagest*, I 1, p. 4.7-9 Heiberg and J. Feke, "Ptolemy's Defense of Theoretical Philosophy", *Apeiron* 45 (2012), pp. 61-90 (p. 62).

[17] See Bernard, "In What Sense" (quoted above, n. 9), p. 99; J. Feke - A. Jones, "Ptolemy", in L.P. Gerson (ed.), *The Cambridge History of Philosophy in Late Antiquity*, 2 vols, Cambridge U.P., Cambridge 2010, vol. 1, pp. 197-209 (pp. 208-9) and A. Bernard, "The Significance of Ptolemy's *Almagest* for its Early Readers," *Revue de Synthèse* 31/4 (2010), pp. 495-521 (p. 502-12).

[18] *Taʿlīmī* translates literally μαθηματικόν in the sense of "subject of instruction". Just as μαθηματική derives from the verb μανθάνειν which means 'to learn', *taʿlīmī* and *taʿālīm* derive from the root *ʿilm* which means 'knowledge'. And just as μάθημα "can be any branch of learning", so is *taʿlīm*. However, from the 5th century BC onward, arithmetic, geometry, harmonics and astronomy came to occupy a privileged position as the μαθήματα par excellence (see G.E.R. Lloyd, "What Was Mathematics in the Ancient World? Greek and Chinese Perspectives", in E. Robson - J. Stedall [eds.], *The Oxford Handbook of the History of Mathematics*, Oxford U.P., Oxford 2009, pp. 7-25 [pp. 8 ff.]). The pre-eminent place given to mathematical subjects in the Pythagorean school and later in the Platonic curriculum for the education "encouraged the habit of speaking of these subjects exclusively as *mathêmata*" (see T. Heath, *A History of Greek Mathematics*, vol. I: *From Thales to Euclid*, The Clarendon Press, Oxford 1921, pp. 10-25 [p. 10]). Al-Kindī uses indifferently, and seems not to distinguish between *ʿilm al-riyāḍiyyāt* (*Fī l-Ṣināʿa l-ʿuẓmā*, p. 118 Aḥmad), *al-riyāḍa* (*ibid.*, p. 119) or *al-ʿilm al-riyāḍī* (*ibid.*, p. 123) on the one hand and *al-ʿulūm al-taʿālimiyya* or *al-taʿālīm* (p. 124; 125; 127) on the other. Therefore and because the "instructional sciences" at stake are all mathematical and cover exclusively subjects that were considered as mathematical, I will translate accordingly both *taʿlīm* and its derivatives as well as *riyāḍī* and its derivatives by "mathematical". When the redundancy of the formulation requires, I will translate *taʿlīmī* by 'instructional'.

[19] Al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, p. 124.3-4 Aḥmad.

subject-matter of each science, i.e. the principles constitutive of the being of all things that exist, that is, according to al-Kindī (following Theon), "of all natural bodies" that are composed of matter, form, and movement. Whereas Ptolemy did indeed link the tripartite division of theoretical sciences to the fact that "everything that exists is composed of matter, form, and motion" he nevertheless did not qualify each science by one principle in particular, as already noticed by J. Feke.[20] Instead he describes theology as concerned with investigating "the first cause of the first motion of the universe [...] [which] can be thought of as an invisible and motionless deity". By contrast, physics "investigates material and ever-moving nature", and mathematics

> can make statements about quality in respect of shapes and in respect of motion from place to place, because it can inquire into shape, quantity, size, and, further, into place, time, and the like. Such being falls, as it were, between those [other] two not only because it can be thought of through sense-perception and apart from sense-perception, but also because [such being] is a property of absolutely all things that are both mortal and immortal.[21]

Therefore when al-Kindī qualifies, in what follows, each science by one principle in particular: "motion being constitutive of the theological science and matter being constitutive of the natural science [...] as for the science to which the name mathematics (*taʿālīm*) specifically applies its cause is the form only",[22] he is rather following Theon's commentary that gives a more systematic turn to Ptolemy's thought here. As pointed by Feke, Ptolemy underscores the opposition perceptibility *vs*. imperceptibility of the object of each science.[23] Such an emphasis is made explicit in the first argument he brings forward (in the passage quoted above) for the ranking of mathematics as intermediate between physics and theology. The same argument is reproduced by al-Kindī when, in the wake of Ptolemy, he ranks mathematics as:

> intermediary between natural science and theology, because natural [science] needs the senses in order to be grasped, whereas theology needs the intellect and the comprehension (*al-fahm*) to be grasped; as for the intermediary science, which is the mathematical science, it is possible to grasp its knowledge (*idrāk al-ʿilm bihi*) with or without the senses, I mean with the intellect.[24]

If the division of the three theoretical sciences rests on an ontological criterion, being determined by the nature of the object of each science, their classification and the intermediary position assigned to mathematics between physics and theology rests first on an epistemological argument: the perceptibility or imperceptibility of the knowledge they yield.[25] Therefore it is a theory of knowledge that is at stake which ultimately aims at promoting mathematics and especially astronomy as the most accurate science, or in other words the one that reflects reality best.[26]

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[20] See J. Feke, "Ptolemy in Philosophical Context. A Study of the Relationships between Physics, Mathematics and Theology," PhD, Institute for the History and Philosophy of Science and Technology, University of Toronto 2009, p. 24.

[21] See Ptolemy, *Almagest*, I 1, pp. 5.25-6.8 Heiberg; translated in A. Bowen, "The Demarcation of Physical Theory and Astronomy by Geminus and Ptolemy", *Perspectives on Science* 15/3 (2007), pp. 327-58 (p. 350); see also Toomer, *Ptolemy's Almagest*, p. 36.

[22] Al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, p. 125.11-15 Aḥmad.

[23] See Feke, "Ptolemy in Philosophical Context" (quoted above, n. 20), pp. 24ff.

[24] Al-Kindī, *Fī l-Ṣināʿaa l-ʿuẓmā*, p. 126.6-9 Aḥmad.

[25] See Feke, "Ptolemy in Philosophical Context" (quoted above, n. 20), pp. 23-39.

[26] See Bowen, "The Demarcation of Physical Theory and Astronomy" (quoted above, n. 21), p. 352.

The appropriation of Ptolemy's division of theoretical sciences by al-Kindī raises several issues that this paper plans to address. First the classification of mathematics as an intermediary science between theology, on the one hand, and physics on the other, that is reiterated in other treatises of his[27] can be now safely traced back to Ptolemy's philosophical preface to his *Almagest*, as already noticed by Rosenthal.[28] It is there, that al-Kindī finds also an intellectual program that he will apply throughout his work and that is reproduced in the prologues of several of his treatises.[29]

Having said that, in his introductory *Epistle* to Aristotle's philosophy, *On the Quantity of Aristotle's Books and What is Necessary for the Attainment of Philosophy*, al-Kindī obviously seems to consider mathematics as a propaedeutic to the study of philosophy, stating from the outset that "the knowledge of Aristotle's books should be acquired after the knowledge of mathematics".[30] Moreover, the division of Aristotle's books into four categories, that he introduced in this same treatise, includes in the third and hence intermediary category books that study "the things that don't need bodies and are not connected with bodies in order to subsist and persist, but can exist with bodies" (p. 368) under which he lists next to the *De Anima*, three of the *Parva naturalia* (*De Sensu et sensibilibus*; *De Somno et vigilia*; *De Longitudine et brevitate vitae*). All four books fall under the header of "psychology" which can thus be considered as an intermediary science. It is worth noting that such a classification rests on the ontological criterion of separability which was used by Aristotle in his own classification of knowledge and according to which he characterized mathematical objects, with some hesitation, as "immovable but probably not separable, but embodied in matter". A few lines before, though, he had recognized that "it is not clear" whether mathematical objects "are immovable and separable from matter", "it is clear however that, it [i.e. mathematics] considers some mathematical objects qua immovable and qua separable from matter" (*Metaph.* E 1, 1026 a 7 ff., tr. Barnes).[31]

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[27] See al-Kindī's treatise *On The String Instruments Producing Sound, From the One String* [*Instrument*] *to the Ten Strings* [*Instrument*] (*Kitāb al-Muṣawwitāt al-watariyya min ḏāt al-watar al-wāḥid ilā ḏāt al-ʿašarat awtār*) in *Muʾallafāt al-Kindī al-Mūsiqiyya*, ed. Z. Yūsuf, Maṭbaʿat Šafīq, Bagdad 1962, pp. 69-92, pp. 70ff. and G. Endress, "Mathematics and Philosophy in Medieval Islam", in J.P. Hogendijk - A.I. Sabra (eds.), *The Enterprise of Science in Islam. New Perspectives*, MIT Press, Cambridge Mass. - London 2003, pp. 121-76, at p. 130. See also *On the Explanation of the Finitude of the Universe* (*Fī Īḍāḥ tanāhī ǧirm al-ʿālam*), in Rashed - Jolivet, *Œuvres Philosophiques* (quoted above, n. 3), vol. II, p. 165, quoted by D. Gutas, "Geometry and the Rebirth of Philosophy in Arabic with al-Kindī", in R. Arnzen - J. Thielmann (eds.), *Words, Texts and Concepts Cruising the Mediterranean Sea. Studies on the Sources, Contents and Influences of Islamic Civilization and Arabic Philosophy and Science Dedicated to Gerhard Endress on his Sixty-Fifth Birthday*, Peeters, Leuven 2003 (Orientalia Lovaniensia Analecta, 139), pp. 195-209, at p. 204.

[28] See F. Rosenthal, "From Arabic Books and Manuscripts VI: Istanbul Materials for al-Kindī and as-Saraḫsī," *Journal of the American Oriental Society* 76/1 (1956), pp. 27-31 (pp. 28-9).

[29] See Rosenthal, "Al-Kindī and Ptolemy" (quoted above, n. 1), pp. 445-7.

[30] See *Risālat al-Kindī Fī Kammiyat kutub Arisṭūṭālīs wa-mā yuḥtāǧu ilayhi fī taḥṣīl al-falsafa*, in *Rasāʾil al-Kindī al-falsafiyya*, vol. I, pp. 363-84 Abū Rīda (p. 364; p. 369; p. 370).

[31] Unless otherwise specified, all translations of Aristotle's works are cited from *The Complete Works of Aristotle. The Revised Oxford Translation*, ed. J. Barnes, 2 vols, Princeton U.P., Princeton 1984. See G. Endress, "The Circle of al-Kindī. Early Arabic Translations from the Greek and the Rise of Islamic Philosophy", in G. Endress - R. Kruk (eds.), *The Ancient Tradition in Christian and Islamic Hellenism. Studies on the Transmission of Greek Philosophy and Sciences dedicated to H.J. Drossaart Lulofs on his Ninetieth Birthday*, Research School CNWS, Leiden 1997 (Archivum Graeco-Arabicum, 1), pp. 43-76 (p. 64) who draws attention to the parallelism between the "soul as subject and the mathematicals as object" which both belong "to the same intermediate realm 'in between'" the eternal intelligibles and the corruptible sensibles. See also Id., "al-Kindī über die Wiedererinnerung der Seele: Arabischer Platonismus und die Legitimation der Wissenschaften im Islam," *Oriens* 34 (1994), pp. 174-221 (pp. 181-2). I owe the last reference to the anonymous reader of this article.

The question remains as to whether all these passages are in blatant contradiction with each other. Is al-Kindī inconsistent in his division of theoretical sciences, sometimes considering mathematics as an intermediary science and sometimes as a propaedeutic to the study of philosophy while psychology is assigned the intermediary rank?[32]

*I. Mathematics as an intermediary theoretical science*

The tension between the intermediate character of mathematics as a theoretical science and the priority that it is granted as a propaedeutic to the acquisition of knowledge is already found in *On the Great Art* where both aspects are intricately entangled from the beginning of the treatise. Indeed, al-Kindī opens his prologue by immediately absolving Ptolemy (*wāḍiʿ hāḏa al-kitāb*) of any responsibility in the opacity of his book, which is due mainly "to the elevation of this art, the nobility of its ranking and the necessity, for those who examine it theoretically, to use as a propaedeutic to science (*taqdīm al-ʿilm*) two arts of the mathematical science (*ʿilm al-riyāḍiyyāt*), namely, arithmetic and geometry and to delve into physics (natural science) as well as metaphysics".[33] The tone is thus set from the beginning and will be re-emphasized a few lines later:

> The man who designed [this art] (i.e. Ptolemy) did not establish it for the students that are beginners in theory, but for those who are advanced in knowledge (science) (*li-allaḏīn qad ʿalū fī l-ʿilm*). This is what he said in his book on [this art].[34]

Actually, al-Kindī is expanding on a remark made by Ptolemy at the end of his preface, according to which he assumes that his readers have already some competence. He characterizes them as "those who have already made some progress" (*Alm*. I, 8.8-9 Heiberg), hence he will try to be as concise as possible. As correctly understood by al-Kindī, this is obviously an allusion to the geometrical and arithmetical preliminaries necessary for the study of advanced astronomy. This means that Ptolemy assumed a knowledge of elementary geometry and arithmetic from his reader in addition to "spherics".[35]

Al-Kindī emphasizes further the role of the propaedeutic sciences by reminding us of the curriculum the student needs to follow prior to the study of astronomy and to which he has himself contributed a list of books which "he has written as an introduction" to astronomy (*fātiḥ lahā*).

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[32] See Endress, "Mathematics and Philosophy" (quoted above, n. 27), pp. 129-30 and, P. Adamson, *al-Kindī*, Oxford U.P. Oxford 2007 (Great Medieval Thinkers), pp. 31ff. for a clear presentation of the terms of the issue. For a discussion of mathematics and philosophy in al-Kindī, see R. Rashed, "Al-Kindi's Commentary on Archimedes' *The Measurement of the Circle*", *Arabic Sciences and Philosophy* 3 (1993), pp. 7-53 (pp. 7-12); Endress, "Mathematics and Philosophy" (quoted above, n. 27), pp. 127-31 and Id., "Building the Library of Arabic Philosophy. Platonism and Aristotelianism in the Sources of al-Kindī", in C. D'Ancona (ed.), *The Libraries of the Neoplatonists. Proceedings of the Meeting of the European Science Foundation Network "Late Antiquity and Arabic Thought. Patterns in the Constitution of European Culture" (Strasbourg, March 12-14 2004)*, Brill, Leiden 2007 (Philosophia Antiqua, 107), pp. 319-50 (pp. 338-44); Gutas, "Geometry and the Rebirth of Philosophy" (quoted above, n. 27), pp. 201-9; Id., "Origins in Baghdad", in R. Pasnau - Ch. van Dyke (eds.), *The Cambridge History of Medieval Philosophy*, 2 vols, Cambridge U.P., Cambridge 2010, pp. 11-25 (pp. 21-23); Adamson, *al-Kindī*, pp. 30-8 and Ch. 7.

[33] Al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, p. 118.11-13 Aḥmad.

[34] *Ibid.*, p. 120.1-2.

[35] See Toomer, *Ptolemy's Almagest*, p. 37 and "Introduction", p. 6, and A. Bernard, "The Alexandrian School. Theon of Alexandria and Hypatia", in Gerson (ed.), *The Cambridge History of Philosophy in Late Antiquity* (quoted above, n. 17), pp. 417-36 (p. 427).

The list of books al-Kindī enumerates[36] seems to constitute an organized curriculum starting with elementary treatises on geometry and arithmetic, and followed by those "which are ranked after the *Book of the Elements of Geometry*" (*murrataba … baʿd Kitāb al-Usṭuqussāt fī l-misāḥa*), that is the books that should be studied after Euclid's *Elements* and before Ptolemy's *Almagest*. These include, among others, a *Book on the Motion of the Sphere* (*K. Fī Ḥarakat al-kura*), a *Book on Optics* (*K. Fī l-Manāẓir*) and a *Book on the Habitable Places* (*K. Fī l-Masākin*).[37]

Next, following his source, but expanding on it at great length, al-Kindī distinguishes practical and theoretical philosophy and contends that we have "to seek human perfection in each one of them". That means, as far as theoretical philosophy is concerned, "to increase our devotion and extend our preoccupation in teaching the mathematical science (*taʿlīm al-ʿilm al-riyāḍī*) to which the name science applies specifically".[38] This is a slight shift from Ptolemy's text which affirms that he "thought it fitting to […] devote most of [his] time to intellectual matters, in order to teach theories (θεωρήματα) which are so many and beautiful, especially those to which the epithet 'mathematical' is particularly applied",[39] unless we follow A. Bowen's translation of μαθηματικός (-ή -όν) by "scientific".[40] Al-Kindī's paraphrase becomes then an explanation of what the theoretical studies "to which the epithet 'scientific' (μαθηματικόν) is particularly applied" are, namely the mathematical sciences that, following Theon's commentary, al-Kindī enumerates, immediately after, as being the four μαθήματα of the *quadrivium*:

> I mean by the instructional sciences (*al-ʿulūm al-taʿālimiyya*), arithmetic, geometry, astronomy and harmonics. These four instructional [sciences] are the divisions of the mathematical science (*al-ʿilm al-riyāḍī*) which is the path towards the theoretical science.[41]

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36 See al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, p. 120.5-14 Aḥmad.

37 These books are not labeled by al-Kindī as *mutawassiṭāt* (intermediary), and are said to have been composed by himself. Whether the program he draws is modeled after an existing curriculum is hard to say, as it seems that so far there is no documented evidence (preserved papyri) attesting the existence of an organized curriculum leading the student progressively from elementary treatises like Euclid's *Elements* to the more advanced ones like the *Almagest* (see Bernard, "In What Sense" [quoted above, n. 9], p. 106, n. 57). Having said that, the titles that al-Kindī lists correspond to a great extent to the collection known as *kutub al-mutawassiṭāt* that were said to be read before Ptolemy's *Almagest* and after Euclid's *Elements*. See E. Kheirandish, "Organizing Scientific Knowledge. The 'Mixed' Sciences in Early Classifications", in G. Endress (ed.), *Organizing Knowledge. Encyclopaedic Activities in the Pre-Eighteenth Century Islamic World*, Brill, Leiden 2006 (Islamic Philosophy, Theology and Science. Texts and Studies, 61), pp. 135-54 (p. 139) who refers to M. Steinschneider, "Die 'mittleren' Bücher der Araber und ihre Bearbeiter", *Zeitschrift für Mathematik und Physik* (1865), pp. 456-98. I owe this reference to the anonymous reader of this paper.

38 Al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, p. 123.15-16 Aḥmad.

39 Ptolemy, *Almagest*, I 1, p. 5.4-7 Heiberg, tr. Toomer, *Ptolemy's Almagest*, p. 35.

40 See Bowen, "The Demarcation of Physical Theory and Astronomy" (quoted above, n. 21), p. 349 and n. 70. It is worth noting that the paraphrase properly said of Ptolemy's preface, that follows immediately al-Kindī's own prologue, reflects the same ambiguity, since it reads: "to devote most of our preoccupation to learning (*taʿallum*) the great and momentous science (*al-ʿilm al-kabīr al-ḫaṭīr*), and particularly the one to which the name science specifically applies (*al-maḫṣūṣ bi-ism al-ʿilm*)" (see al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, p. 131.7-8 Aḥmad). The redundancy is already found in the Arabic version of al-Ḥaǧǧāǧ ibn Yūsuf ibn Maṭar al-Ḥāsib (*fl.* 786-830) and Sarǧūn ibn Hilyā al-Rūmī: *wa-an nabḏula akṯar farāġinā wa-naǧʿala akṯar ʿināyatinā fī taʿallum al-ʿilm al-kabīr al-ḫaṭīr wa-ḫāṣatan al-maḫṣūṣ bi-ism al-ʿilm* (ms. Leiden, *Or.* 680, f. 2v23-24). Al-Kindī's paraphrase reproduces almost literally the same formulation with a slight permutation in the order of the words. See also ms. London, British Library, *Add.* 7474, f. 1r 13-14.

41 Al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, pp. 123.17-124.2 Aḥmad (slightly amended).

However, al-Kindī proceeds immediately to specify that "the theoretical science is [in turn] divided into three divisions: one of them is natural science, the other is the mathematical science (*al-ʿilm al-taʿlīmī*) while the third is theology". Arithmetic, geometry, astronomy and harmonics are thus part of the mathematical division of theoretical philosophy.[42] At the same time they are "the path towards the theoretical science". How can the instructional sciences be the "path", that is, the method that leads to the theoretical science, and at the same time one of the theoretical sciences? Ultimately it is the status of mathematics which is at stake: is it an *organon* or is it part of theoretical philosophy?

### *I.i Mathematics as the Only Science that Yields Knowledge*

First, it is worth noting that Ptolemy does not mention the *quadrivium* in his *Preface*. He always refers to the mathematical science as τὸ μαθηματικόν and defines it according to its object as: "The kind of theoretical philosophy which can make statements about quality in respect of shapes and in respect of motions from place to place, because it can inquire into shape, quantity, size, and, further into place, time and the like", as already stated above.[43] Mathematics studies the mathematical properties of physical objects, qua mathematical, meaning in abstraction from the physical substrate in which they are embedded. For, as stated by Aristotle, when he was drawing the boundary between physics and mathematics, "natural bodies contain surfaces and volumes, lines and points and these are the subject matter of mathematics" (*Phys.* II 2, 193 b 24-25). However, the mathematical models drawn by Ptolemy rest on certain physical assumptions.[44] Astronomy does not exclude physical investigations, as it is concerned with the arrangement of celestial bodies, the shapes, magnitudes and distances of the earth, the sun and the moon, as well as with the conjunctions of the planets and the quantitative and qualitative properties of their movements.[45] Dealing with shapes from the point of view of quality, magnitude and quantity, astronomy thus naturally relies on arithmetic and geometry. Hence Ptolemy can accordingly rank mathematics as an intermediary science between physics and theology as its object can be conceived through sense perception (the visible aetherial bodies) and apart from sense perception (magnitudes, lines, numbers etc.).

Having said that, al-Kindī's description of mathematics as a theoretical science provided a few lines below, following in the footsteps of Ptolemy, leaves little doubt as to the fact that the four 'instructional' sciences not only count as branches of mathematics but also constitute the reason why mathematics is the only science that produces sure and unshakable knowledge rather than conjecture, since it proceeds "by truthful methods (*al-ṭuruq al-ḥaqīqiyya*), namely the geometrical proofs and arithmetic that are incontrovertible".

> As for the science to which the name mathematics (*ism al-taʿālīm*) applies specifically,[46] its principle[47] is only the form. The quality (nature) of mathematics (*kayfiyyat al-taʿālīm*) is determined by the form

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[42] See Bowen, "The Demarcation of Physical Theory and Astronomy" (quoted above, n. 21), p. 349, n. 70.

[43] Ptolemy, *Almagest*, I 1, pp. 5.25-6.8 Heiberg; tr. Bowen, "The Demarcation of Physical Theory and Astronomy" (quoted above, n. 21), p. 350; see also Toomer, *Ptolemy's Almagest*, p. 36 and above, p. 87.

[44] See G.E.R. Lloyd, "Saving the Appearances", *Classical Quarterly* 28 (1978), pp. 202-22, repr. in Id., *Methods and Problems in Greek Science. Selected Articles*, Cambridge U.P., Cambridge 1993$^2$, pp. 248-77 (p. 250), from which this paragraph draws.

[45] See Simpl., *In Phys.* 2, pp. 290-92 Diels (*CAG* IX).

[46] The edited text as well as the manuscript have here: *wa-innamā* [*ism*] *al-ʿilm al-maḥṣūṣ bi-ism al-taʿālīm*. I have opted to drop the first occurrence of *ism* which is obviously added mistakenly.

[47] Lit. "its cause" (*sababahu*), however *sabab* refers here to the constitutive principle of the mathematical science, that is the form.

and the like and also by the movements from place to place (*ḥarakāt al-intiqāl*). I mean by form here, what exists in matter, namely the surface and the limit. I mean by the quality of the form, the figure, like the triangle, the square and the like. It [i.e. the nature of mathematics] is also made evident through magnitude (*al-ʿiẓam*) to which measures apply, and through full quantity, such as number, time and place that are sought in astronomy, because astronomy includes the knowledge of position and time. Concerning the stars (i.e. the planets), don't we seek their position and how long it takes for them to complete their revolutions?

[...] It thus happens that this science concerns everything that comes to be, be it mortal or immortal. Everything that comes to be has a part, a limit and a figure. I mean by [the things] that come to be, the natural bodies concerning which we already said that they are produced of matter, form and movement. If the universe (*al-kull*) has a limit (*nihāya*), an extremity (*ṭaraf*) and a figure (*šakl*), and the kind of mathematical [science] comprehends (*muḥīṭ*) them, then mathematics comprehends the universe. As for the ever changing [bodies] – I mean those which are in the changing nature which undergoes corruption – [mathematics is concerned with the forms that are][48] with them and are inseparable from them, because this nature always comes to be with a form and a surface and that kind is always changing with movement. As for the eternal aetherial [bodies] (*al-abadiyya al-aṯīriyya*) which are the heavens and the heavenly [bodies], the mathematical kind of science is concerned with them as well since they also have a form.

The form which is in the celestial aetherial nature is motionless, that is why we have given precedence to the mathematical kind over the kinds of the other two sciences, and also because we observe that natural [science] is incomprehensible (*ġayr muḥāṭ bihi*) due to the flux (*sayalān*) of matter and the rapidity of its change: it is indeed different in everything (*fa-huwa fī kullin āḫar*).[49] As for the divine [kind] it is ungraspable by a science that would comprehend it. Therefore and because the [divine being] is not manifest to any of the senses and is not connected to the sensible [things] but is ever separated from them, as we have shown in our book *On The First* [...](?) *Philosophy*[50] (*Fī l-Falsafa l-ūlā l-dāḫila*), only His actions are perceptible, on account of which the knowledge of Whom (*maʿrifatuhu*), great be His praise, follows necessarily, and the human intellects have submitted to acknowledge Him.

[48] There is a blank left in the manuscript at this point that could be filled by one or two words. The words between brackets are thus my addition.

[49] I have disregarded the emendation suggested by the editor here, even though it fits in with the general meaning of the sentence: *fa-huwa fī kulli* [*waqt šayʾ*] *āḫar*.

[50] In the bibliographical lists of al-Kindī's works a *Kitāb al-Falsafa al-dāḫila wa-l-masāʾil al-manṭiqiyya wa-l-muʿtāṣa wa-mā fawq al-ṭabīʿiyyāt* (*Book of* [...] (?) *Philosophy and of the logical and difficult questions and what is above physics*) is listed next to the *Kitāb al-Falsafa l-ūlā fī-mā dūn al-ṭabiʿiyyāt wa-l-tawḥīd* (*Book of First Philosophy, on what is beyond physics, and of Oneness*), see Ibn al-Nadīm, al-*Fihrist*, vol. I, p. 255 Flügel, followed by al-Qifṭī, *Taʾrīḫ al-ḥukamāʾ*, ed. J. Lippert, Dieterich, Leipzig 1903, p. 368 and Ibn Abī Uṣaybiʿa, *ʿUyūn al-anbāʾ fī ṭabaqāt al-aṭibbāʾ*, ed. A. Müller, 2 vols, Selbstverlag, Könisberg - Le Caire 1882, vol. I, p. 206). The first treatise did not come down to us but the phrase "*al-falsafa l-dāḫila*" occurs in *K. al-Imtāʿ wa-l-muʾānasa* in an ambiguous context, where al-Kindī is put to test and fails to recognize that the philosophical questions by which he has been confuted were taken from "*al-falsafa l-dāḫila*". It is not very clear whether the denomination refers to a title or to a genre (see Abū Ḥayyān al-Tawḥīdī, *Kitāb al-Imtāʿ wa-l-muʾānasa*, ed. A. Amīn - A. al-Zayn, 3 vols in 1, al-Maktaba al-ʿaṣriyya, Beirut - Saida 1953, vol. I, p. 127). Whether our text refers to the first treatise or the second is hard to tell, all the more that even though the theme of the absolute transcendence and hence separateness of God is present in *On First Philosophy*, the issue of God's agency in the world has not been addressed in the part that has reached us. See Rosenthal, "al-Kindī and Ptolemy" (quoted above, n. 1), p. 442 who suggests either the possibility of a combined edition of both treatises to which al-Kindī might be referring, or the assumption of an erroneous reference.

> As for the mathematical kind of science, the way to grasp it is through the true methods I have made clear, namely the geometrical demonstrations (*al-barāhīn al-misāḥiyya*) and arithmetic which are incontrovertible. Therefore we should exert all efforts in dedicating ourselves to seeking the knowledge of all kinds of mathematics, especially astronomy for that alone is perpetually devoted to the eternally arranged and preeminent [bodies], and it is eternally stable in the same state.[51]

Following closely his two sources, namely Ptolemy and his commentator Theon, and reflecting their eclecticism, al-Kindī singles out mathematics as the only one among the three theoretical sciences that yields knowledge due to the nature of its object. Physics and theology are conjectural because the former deals with unstable and perpetually changing material that cannot yield stable and sound inferences, whereas the latter is concerned with the unmoved mover of "the first motion of the universe", which is motionless but unseen (*lā yurā*) and hence ungraspable.[52] Mathematics deals with forms,[53] from the permanence and stability of which it draws its accuracy, and with motion from place to place which is considered by Aristotle as the primary kind of motion and as being the only possible motion for eternal things.[54] The forms at stake are of course the forms mentally separated from the matter in which they are actually embedded. Shapes, sizes and limits, if considered with respect to magnitude, fall under geometry; with respect to quantity they fall under arithmetic, and under astronomy with respect to time and place. Studying the properties of the sublunar as well as the heavenly bodies, mathematics thus encompasses everything in the universe. However it owes its epistemological primacy, not to its all-encompassing property, but to the inalterable and changeless form of the aetherial heavenly bodies, the study of which falls within its province, knowing that the aether does not undergo generation and corruption but moves eternally with a uniform circular motion.

Hence the epistemological nature of mathematics is determined by the ontological status of its loftier objects,[55] that is the heavenly bodies. Beyond the Platonic inspiration of such a statement, what is important is that Ptolemy has demoted theology to the benefit of mathematics. Astronomy studies mathematical objects that are divine, eternal and unchanging, whereas theology is confined to the study of the invisible, or in other words imperceptible, eternal unmoved mover. Such downgrading is ironically in keeping with the Aristotelian assumption that knowledge rests ultimately on perception.[56] Perception as a faculty of discrimination originates knowledge: even though there is no science of the particular that can only be known through sensation, yet "perception is of the universal" (*An. Post.* II 19, 100 a 17). Mathematical objects are no doubt

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[51] Al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, pp. 125.17-127.15 Aḥmad.

[52] See Feke, "Ptolemy in Philosophical Context" (quoted above, n. 20), p. 54.

[53] See Arist., *An. Post.*, I 13, 79 a 7: "Mathematics is about forms, for its objects are not said of any underlying subject – for even if geometrical objects are said of some underlying subject, still it is not as being said of an underlying subject that they are studied" (tr. Barnes). G.G. Granger, *La Théorie aristotélicienne de la science*, Aubier, Paris 1976 (Analyse et raisons), p. 302 points to the incomparable accuracy of mathematical science due to the abstract character of its object; see also *An. Post*. I 27, 87 a 31-37.

[54] See Arist., *Phys.* VIII 7.

[55] The statement is definitely of Platonic inspiration, as has been already noticed by Feke, "Ptolemy in Philosophical Context" (quoted above, n. 20), pp. 43 ff.

[56] See Arist., *An. Post.,* I 18, that starts with the well-known statement: "that if some perception is wanting, it is necessary for some understanding to be wanting too"; and Granger, *La Théorie aristotélicienne* (quoted above, n. 53), pp. 31 ff., who emphasizes the relationship between science and sensation through induction. The objects of scientific knowledge can only be reached through *sensibilia*.

immutable and abstract but ultimately they are abstracted by the νόησις from the sensible in which they reside in potentiality, and hence their object is prior to the φύσεις with respect to the concept.[57] By contrast, the object of theology is radically separated from the sensible, and hence "is ungraspable by a science that would comprehend it", as emphasized by al-Kindī who dealt extensively with the nature of the true, transcendent and almost ineffable One in his own treatise *On First Philosophy*, regardless of whether it is to this particular work that he is referring in the passage quoted above.[58]

### *I.ii Mathematics between Perception and Reason*

The same remark occurs in his treatise *On the String Instruments*, where the classification of mathematics, in the middle between physics and metaphysics, is also detailed in terms of the *quadrivium* and connected with the division of philosophy between theory and practice, though al-Kindī uses there the dichotomy *ʿilm* (knowledge) *vs. ʿamal* (practice) rather than *naẓar vs. fiʿl* which are the terms used in the prologue of *On the Great Art*.[59] The interplay between perception and reason proper to mathematics,[60] between a "science of nature" and a science of "what does not belong to nature but whose effect is observable in the nature" is clearly emphasized.

> **It was a habit of the philosophers to practice the intermediary science** (*al-irtiyāḍ bi-l-ʿilm al-awsaṭ*),[61] [ranked] between a science beneath it and a science above it. The one beneath is the science of nature and of that which is affected by nature (*mā yanṭabiʿu ʿanhā*), whereas the one above is called the science of what does not belong to nature – albeit its effect (*aṯaruhu*) is observed in nature. The intermediary science which leads the way to (*yatasabbal*)[62] the science of what is above it and of what is below it, is divided into four divisions, namely: 1) the science of number and what is numbered (*al-ʿadad wa-l-maʿdūdāt*), that is arithmetic; 2) the science of harmony (*ʿilm al-taʾlīf*), that is music (*al-mūsīqā*); 3) the science of *ǧaʾūmaṭriya*, that is geometry (*al-handasa*); and 4) the science of *asṭrunūmīya*, that is astronomy (*al-tanǧīm*). The [philosophers] used to consider championing it (i.e. the intermediary science) in their discourses and making it manifest to the senses, so that the intellects of those [endowed] with intelligent natures (*ḏawī l-fiṭan al-ḏakiyya*) would testify to the veracity of their accounts. Those who ignored the precedence of knowledge (*ʿilm*) over practice (*ʿamal*) are those who did not learn that

[57] See Arist., *Metaph.*, VI 1, 1026 a 15 and Granger, *La Théorie aristotélicienne* (quoted above, n. 53), pp. 295 ff.

[58] See particularly *On First Philosophy*, Chap. 3 and 4.

[59] Al-Kindī, *Fī l-Ṣināʿa al-ʿuẓmā*, p. 121.14-15 Aḥmad. On the usage of the dichotomy *ʿilm vs. ʿamal* in the division between theoretical and practical philosophy see Qusṭā ibn Lūqā's treatise on the classification of sciences, which includes a division of theoretical philosophy where mathematics is also located in the middle between physics and theology and labeled as *al-ʿilm al-awsaṭ*, quoted by Kheirandish, "Organizing Scientific Knowledge," p. 149. On the striking similarities between this treatise and al-Kindī's work see H. Daiber, "Qosṭā ibn Lūqā (9. Jh.) über die Einteilung der der Wissenschaften", *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften* 6 (1990), pp. 93-129 (pp. 108-10) which includes a critical edition, a German translation and a commentary of Qusṭā's treatise.

[60] See Feke, "Ptolemy in Philosophical Context" (quoted above, n. 20), pp. 56ff. who analyzes Ptolemy's scientific method as portrayed in the *Almagest*, the *Kriterion*, and the *Harmonics* as an interplay between reason and sense perception "and aimed at the construction of knowledge".

[61] Interestingly, al-Kindī does not label any of the three sciences, but identifies them only by their different objects. This being said, *al-irtiyāḍ* derives from the same root as *al-riyāḍa* (pl. *al-riyāḍiyyāt*), which is one of the names of the mathematical science.

[62] I follow here Endress' translation; Endress reproduces this paragraph partially in his article "Mathematics and Philosophy" (quoted above, n. 27), p. 130.

the knowledge of everything precedes its action (*fiʿlahu*) and that there is no way to make manifest something correctly before it becomes stable in the mind as an object of knowledge, and in nature as an object of comprehension, and in the soul as an object of intellection, and then becomes manifest as intelligible, sensible, corporeal (*muǧassam*), and tangible.[63]

This passage presents some striking similarities with the prologue of *On the Great Art* and by the same token with the Preface to the *Almagest*. Mathematical science appears here clearly as the path between the science of nature, on the one hand, and the "science of what does not belong to nature", on the other, but whose effects are observed in nature and whose existence can thus only be deduced logically although it remains imperceptible and invisible. The statement echoes the description of mathematics in *On the Great Art*, between natural science and the divine transcendent being "ungraspable by a science" but accessible only through His actions in the world.[64] The usage of the 5th form of the verb *sabala*, i.e. *tasabbala*, is indicative of the active and crucial role attributed to mathematics that are not only ranked in between the other two sciences but "lead the way" towards both of them. In other words, the science of nature and "the science of what does not belong to nature" can be reached through the mathematical science that appears here to be proper to the philosophers or, more precisely, to be the practice they are accustomed to (*ʿādat al-falāsifa*). A few lines after that, the very same formulation is used in order to explain the activity of the philosophers as "making manifest the secrets of the science of nature and its effects".

We find he who did something (*ʿamala šayʾan*) without knowledge and deep reflection – and the same [situation] exists in nature – not knowing whether that [action] was correct or wrong; if it happens that he acted correctly, he does not know the reason why (*al-ʿilla*) it is correct. If asked about it he would not succeed in explaining it with a proof (*ḥuǧǧa*) that he could express by himself. Therefore he who is in that situation cannot be praised, because he does not know the cause/reason of what he crafted (?) (*fī-mā ṣanaʿa*).[65] **It was the habit of the philosophers to make manifest the secrets of the science of nature** and its effects in many of the subjects [they have treated] in their books. Among them are those they have called[66] *On Arithmetic and the Amicable and Inimical Numbers* (*al-aʿdād*

[63] See *On the String Instruments*, in *Muʾallafāt al-Kindī al-Mūsiqiyya*, pp. 70.13-71.4 Yūsuf, and Endress, "Mathematics and Philosophy" (quoted above, n. 27), pp. 130-1 for a partial translation.

[64] See above, p. 92.

[65] A similar account can be found in *On the Great Art*, echoing the very first lines of *Almagest* 1.1 that states that "practical philosophy, before it is practical, turns out to be theoretical": "It happens that action (*al-fiʿl*) becomes theory (*naẓar*) for when the philosophers (*ḏawī l-ḥikma*, lit. those endowed with wisdom) intend to do something they precede it with a theoretical exam and an investigation of the knowledge of what needs to be done. The knowledge of what needs to be done must come first, since the aim is to pick up the praiseworthy choice (*al-muḫtār al-aḥmad*)"; see al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, p. 122.4-7 Aḥmad.

[66] It is difficult to determine whether the following are book subjects or book titles. According to J.P. Hogendijk, the earliest known reference to a pair of amicable numbers is in Iamblichus' commentary to Nicomachus, *Introduction to Arithmetic*, although he assumes that the pair was known before his time. Ṯābit ibn Qurra's (836-901) *Treatise On the Derivation of the Amicable Numbers in an Easy Way* constitutes the next major step in the theory of amicable numbers. In the prologue of his treatise, Ṯābit contends that amicable numbers were known to the Pyhtagoreans. However neither Nicomachus, nor Euclid mentioned them or exhibited some interest in them. Therefore, it was Ṯābit who provided a rule for the derivation of amicable numbers along with a proof. See *Kitāb al-Aʿdād al-mutaḥābba li-Ṯābit b. Qurra*, ed. A.S. Saidan, Publication sponsored by the University of Jordan, Amman 1977, p. 33 and J.P. Hogendijk, "Thābit ibn Qurra and the Pair of Amicable Numbers 17296, 18416", *Historia Mathematica* 12 (1985), pp. 269-73; S. Brentjes - J.P. Hogendijk, "Notes on Thābit ibn Qurra and his Rule for Amicable Numbers", *Historia Mathematica* 16 (1989),

*al-mutaḥābba wa-l-mutabāġiḍa*), *On the Proportional Lines* (*al-ḫuṭūṭ al-mutanāsiba*), and *On the Five Solids* (*polyhedra*) *Inscribed in the Sphere*.[67]

In light of the exact similarity between the two formulations (in bold) it is very tempting to consider that "the practice of the intermediary science" would consist in "making manifest the secrets of the science of nature and its effects". In other words, the activity of the philosopher is to bring to light or to make accessible to the mind the intelligible structures or the reason (λόγος) behind the raw data produced by the sense-perception, through mathematical procedures, namely the four branches of mathematics enumerated here in the Pythagorean order that ranks harmonics second after arithmetic and before geometry.[68] Such interaction between perception and reason seems to be the main point of the argument carried on by al-Kindī concerning the precedence of knowledge over action. It emphasizes the necessity to grasp the reason or the rational principles that account for the intelligibility of the object of perception, before "it becomes manifest as intelligible, sensible, solid, and tangible". This means for example that the analysis by abstraction of a body will lead to the position of the solid and its properties; that the analysis by abstraction of movement leads to the position of number etc.[69] For as stated by Ptolemy in the opening lines of the *Harmonics*:

> In everything it is the proper task of the theoretical scientist to show that the works of nature are crafted with reason and with an orderly cause, and that nothing is produced by nature at random or just anyhow, especially in its most beautiful constructions, the kinds that belong to the more rational of the senses, sight and hearing (p. 5.19-23 Düring).[70]

In *On the String Instruments*, the emphasis is thus on the capacity of mathematics to provide knowledge because it is the only science able to give the reason why (*al-ʿilla*) whether in the realm of nature or in the realm of action as stated by al-Kindī in the passage quoted above. The topics of the books listed by al-Kindī as those in which the philosophers used to unearth "the secrets of the science of nature and its effects" are explicit enough in that regard, and leave little doubt as to the fact that the science in charge of providing such knowledge is mathematics and particularly geometry and arithmetic.

In *On the Great Art*, the absolute precedence given to mathematics over the other two theoretical sciences in terms of yielding knowledge is not only due to the conjectural nature of the object of the other two sciences, but also to its "true methods" (*al-ṭuruq al-ḥaqīqiyya*), "namely the geometrical demonstrations and arithmetic which are incontrovertible (*allaḏān lā šakka fīhimā*)". The statement

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pp. 373-8. "On the proportional lines" may refer to Euclid's *Elements* Bk. V. As for the topic of the "Five polyhedra inscribed in a sphere", Bk. XIII of Euclid's *Elements* proves that no other regular bodies than the five polyhedra are possible, and shows how to inscribe them in a sphere.

67 See *On the String Instruments*, in *Muʾallafāt al-Kindī al-Mūsiqiyya*, p. 71.5-13 Yūsuf; partial tr. in Endress, "Mathematics and Philosophy" (quoted above, n. 27) pp. 130-1.

68 This order is the one attributed by Nicomachus of Gerasa, Theon of Smyrna and Proclus to the Pythagoreans (see Heath, *A History of Greek Mathematics* (quoted above, n. 18), vol. I, p. 12 and Endress, "Mathematics and Philosophy" [quoted above, n. 27], p. 130).

69 See Granger, *La Théorie aristotélicienne* (quoted above, n. 53), p. 302.

70 Quoted and translated by A. Barker, *Scientific Method in Ptolemy's Harmonics*, Cambridge U.P., Cambridge 2001, p. 23. See also J. Solomon, *Ptolemy Harmonics,* Translation and Commentary, Brill, Leiden 2000 (Mnemosyne, Supplements), p. 8. On the Arabic transmission of the *Harmonics*, no Arabic translation of which is mentioned by Ibn al-Nadīm's *Fihrist*, and on the possibility that al-Kindī was aware of it, see Rosenthal, "From Arabic Books and Manuscripts" (quoted above, n. 28), p. 28 n. 15.

echoes Ptolemy's claim in his Preface to the *Almagest*, where he singles out mathematics as being the only one among the theoretical sciences able to "provide sure and incontrovertible knowledge to its devotees, provided that one approaches it rigorously. For its kind of proof proceeds by indisputable methods, namely arithmetic and geometry",[71] whereas theology and physics are only conjectural: theology "because of its utterly non-evident and ungraspable character" and natural philosophy "because of matter's unstable and unclear character". The same claim is also reiterated in *Harmonics* III 3, where the relationship between the four mathematical sciences is described through a metaphor initiated by Archytas and Plato that describes geometry and arithmetic as the "instruments of indisputable authority" both harmonics and astronomy employ.

> Related to sight and to the movements in place of the things that are only seen – that is, the heavenly bodies – is *astronomia*; related to hearing and to the movements in place, once again, of the things that are only heard – that is, sounds – is harmonics. They employ both arithmetic and geometry, as instruments of indisputable authority, to discover the quantity and quality of the primary movements; and they are as it were cousins, born of the sisters, sight and hearing, and brought up by arithmetic and geometry as children most closely related in their stock (p. 94.13-20 Düring).[72]

As already noticed by Barker, arithmetic and geometry are not considered as full-fledged mathematical sciences but are instruments that harmonics and astronomy use or "tutors by which they are trained".[73] Barker underscores the analogy between the role of arithmetic and geometry in the realm of mathematics, on the one hand, and the role of logic in the realm of philosophy on the other: both are instruments or methods of argumentations rather than a branch of knowledge that defines a specific scope. Having said that, the double status of logic, as an *organon* and as part and parcel of philosophy, is an issue that was debated among the ancient commentators.[74] The same seems to apply to mathematics, which can thus be considered as a full-fledged theoretical science when the rules of geometry and arithmetic are applied to concrete quantities and qualities, namely the objects of astronomy and harmonics. Along these lines, it is worth noting that at the beginning of *On the Great Art* al-Kindī describes arithmetic and geometry not only as a propaedeutic to astronomy, but as constitutive of it or more precisely as necessary to its subsistence (*fa-ammā l-ʿadad wa-l-handasa, fa-inna qiwām hāḏihi l-ṣināʿa minhumā*).[75] The same claim is reiterated in *On Quantity*.

*II. Mathematic as a Propaedeutic*

The emphasis on the role of geometry and arithmetic as an *organon* finds an echo in al-Kindī's *Epistle On the Quantity of Aristotle's Books and What is Necessary for the Attainment of Philosophy*, where curiously enough the four mathematical sciences are altogether considered from the outset

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[71] See al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, p. 127.11-12 Aḥmad; Toomer, *Ptolemy's Almagest*, p. 36 and above, p. 91.

[72] Trans. Barker, *Scientific Method* (quoted above, n. 70), p. 266; cf. Plat., *Resp.* 530 D (quoted by Barker n. 6); see also Solomon, *Ptolemy Harmonics* (quoted above, n. 70), p. 142.

[73] Barker, *Scientific Method* (quoted above, n. 70), p. 266.

[74] For the terms of the issue see A. Hasnawi, "L'âge de la démonstration. Logique, science et histoire: al-Fārābī, Avicenne, Avempace, Averroès", in G. Federici Vescovini - A. Hasnawi (eds.), *Circolazione dei saperi nel Mediterraneo. Filosofia e scienze (secoli IX-XVII).* Atti del VII Colloquio Internazionale della Société Internationale d'Histoire des Sciences et de la Philosophie Arabes et Islamiques, Edizioni Cadmo, Firenze 2013, pp. 264-5, who quotes (p. 265) Ammonius (*In An. Pr.*, ed. M. Wallies, Reimer, Berlin 1899 [*CAG* IV.6], pp. 10.38-11.1): logic is an *organon* when it uses "pure rules without things (*pragmata*)"; it is part of philosophy when these rules are applied on concrete things.

[75] See al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, p. 119.1 Aḥmad.

as a propaedeutic to the study of Aristotle's books "that the student needs to address successively following their ranking and their order, so that he becomes a philosopher by such means, after the science of mathematics (*baʿda ʿilm al-riyāḍiyyāt*)".[76] As already pointed out by Jolivet,[77] al-Kindī draws a didactic program that establishes the absolute epistemological precedence of mathematics for he who wants to acquire philosophy, as the title of the *Epistle* alludes to: *The Epistle On the Quantity of Aristotle's Books* and *What is Necessary for the Attainment of Philosophy*. That means that all of Aristotle's books will not be enough, on their own, in order to become a real philosopher, that is to reach the "true natures of things" (*ḥaqāʾiq al-ašyāʾ*),[78] if not preceded by a knowledge of mathematics.

A few lines after that, having enumerated Aristotle's books according to the four categories in which he divided them, al-Kindī reiterates the very same claim concerning the propaedeutic role of mathematics that he now lists in its four branches and on which he expands further:

> This is the number of Aristotle's books that we mentioned above – I mean those I have defined by their names – the knowledge of which the perfect philosopher needs to acquire after the science of mathematics. If someone, lacking the knowledge of mathematics, i.e. arithmetic, geometry, astronomy and harmonics, uses these books for a whole lifetime, he still would not be able to have a full knowledge of any of them, and his effort will not make him gain any [knowledge] other than the account [of what he has read], if he indeed has a good memory. As for grasping their very essence (*ʿilmuhā ʿalā kunhihā*), this will never take place if he lacks the knowledge of mathematics. By mathematics I mean the science of arithmetic, harmonics, geometry and astronomy which is the science of the configuration of the universe (*hayʾat al-kull*), of the number of its bodies in their totality, of their motions and the quantity of their motions, and such things that happen [in the universe]. As for arithmetic, it is evident that it is prior to all of them. Indeed if number is abolished (*urtufiʿa*), the things that are numbered are also abolished.[79]

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[76] See *Fī Kammiyat kutub Arisṭūṭālīs*, in *Rasāʾil al-Kindī al-falsafiyya*, ed. Abū Rīda, p. 364.10-11 and Gutas, "Geometry and the Rebirth of Philosophy" (quoted above, n. 27), pp. 201ff.

[77] For a thorough study of this *Epistle*, that highlights its structure and the main points of the argument at stake, see J. Jolivet, "L'*Épître sur la Quantité des Livres d'Aristote* par al-Kindī (une lecture)", in R. Morelon - A. Hasnawi (eds.), *De Zénon d'Elée à Poincaré. Recueil d'études en hommage à Roshdi Rashed*, Peeters, Louvain - Paris 2004 (Les Cahiers du MIDEO, 1), pp. 664-83 (p. 676) on which the following paragraphs draw but from which they also depart. For edition and commentary see M. Guidi - R. Walzer, "Uno scritto introduttivo allo studio di Aristotele", *Memorie della R. Accademia Nazionale dei Lincei, Classe di Scienze Morali, Storiche e Filologiche*, Serie VI, vol. VI, Fasc. V, Roma 1940, pp. 375-419.

[78] See *Fī Kammiyat kutub Arisṭūṭālīs*, in *Rasāʾil al-Kindī al-falsafiyya*, I p. 364.6 Abū Rīda. The same expression occurs in the definition of philosophy provided by al-Kindī in the opening section of *On First Philosophy*, as "the science of the true nature of things according to man's capacity" (*ʿilm al-ašyāʾ bi-ḥaqāʾiqihā bi-qadar ṭāqat al-insān*) see *Rasāʾil al-Kindī al-falsafiyya*, I p. 97.5 Abū Rīda and Rashed-Jolivet, *Œuvres Philosophiques* (quoted above, n. 3), vol. II, p. 9. Compare with the opening lines of Nicomachus of Gerasa, *Introduction to Arithmetic* that attribute to Pythagoras a definition of philosophy as "the knowledge of the truth which is in the beings" (*Introductionis Arithmeticae Libri II*, ed. R. Hoche, Teubner, Leipzig 1866, Bk I 1, p. 2.8). On this point see below, n. 110.

[79] *Fī Kammiyat kutub Arisṭūṭālīs*, in *Rasāʾil al-Kindī al-falsafiyya*, I, pp. 369.14-370.8 Abū Rīda. See also Gutas, "Geometry and the Rebirth of Philosophy" (quoted above, n. 27), p. 202. Compare the last sentence of the passage with the Hebrew version of the so far lost Arabic translation of Nicomachus of Gerasa's *Introduction to Arithmetic* by Ḥabīb ibn Bahrīz, that is said to have been revised under al-Kindī's supervision (on which see further below, p. 101 and n. 91), which reproduces literally the same statement: "En effet si le nombre disparaît, les [choses] comptées n'ont plus d'existence" (trans. G. Freudenthal - T. Lévy, "De Gérase à Bagdad: Ibn Bahrīz, al-Kindī, et leur recension arabe de l'*Introduction arithmétique* de Nicomaque d'après la version hébraïque de Qalonymos ben Qalonymos d'Arles", in R. Morelon - A. Hasnawi (eds.), *De Zénon d'Elée à Poincaré* [quoted above, n. 77], p. 479-544 [p. 538]).

Al-Kindī's argument unfolds following a clear and quite articulate progression: mathematics is now confirmed in its role as a propaedeutic to the study of Aristotle's books, without which the apprentice philosopher will not be able to grasp their essence or their gist (*kunh*). Without mathematics, philosophy would be nothing other than glossolalia. Then al-Kindī lists the mathematical sciences at stake and, as has been already noticed,[80] enumerates them in two different orders. The second order corresponds to the Pythagorean *quadrivium* that ranks harmonics second rather than geometry –which is the case in the Platonic order followed in the first enumeration, aiming at designing a course of study for the apprentice philosopher. Now if right after the first enumeration, al-Kindī moves to the Pythagorean order (in the passage quoted above), it is because this is the order that makes sense for the usage he intends to make of the four mathematical sciences, and that he makes clear a few lines after, namely a method yielding knowledge of the substance through its two main attributes, that is quantity and quality. Accordingly, arithmetic and harmonics will be associated with quantity and geometry and astronomy with quality. This is why he says: "the mathematics *I mean* are arithmetic, harmonic, etc.", meaning the mathematics *he means for that purpose*, that is "the knowledge of the substance and the attributes of the substance", knowing that "the attributes of the first and separate substances are two, namely quantity and quality".[81]

*II.i Al-Kindī and Nicomachus of Gerasa on the Mathematical Quadrivium*

Actually, the quadripartition of the four branches of mathematics attributed by Nicomachus of Gerasa to the Pythagoreans rests on one unifying principle, that is quantity, as has been pointed out by Ph. Merlan. Quantity is either discrete or continuous. Arithmetic deals with discrete quantities *per se*, whereas harmonics deals with discrete quantities in relation to one another. Continuous quantity or magnitude is either at rest or in motion. Geometry deals with the former and astronomy with the latter.[82] It is likely that al-Kindī had the *Introduction to Arithmetic* in mind (and most probably at hand as we will see below).[83] We have already noticed that his enumeration of the mathematical sciences quoted above ended with an almost literal quote from Nicomachus's *Introduction*, establishing the priority of arithmetic over the three other mathematical sciences.[84] However, when listing again the four mathematical sciences in the same sequence further below, and in order to account for the necessity of mathematics in the achievement of "human science", al-Kindī classifies arithmetic and harmonic as both dealing with quantity, the former with discrete quantities *per se* and the latter with quantities in relation to one another, whereas geometry and astronomy are concerned with quality:

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[80] See Jolivet, "L'*Epître Sur la Quantité*" (quoted above, n. 77), p. 676.

[81] See *Fī Kammiyat kutub Arisṭūṭālīs*, in *Rasāʾil al-Kindī al-falsafiyya*, I, p. 370.4-11 Abū Rīda.

[82] See Ph. Merlan, *From Platonism to Neoplatonism*, M. Nijhoff, The Hague 1975, p. 89; Nicomachus of Gerasa, *Introduction to Arithmetic*, I 3.1-2 Hoche; tr. in Nicomachus of Gerasa, *Introduction to Arithmetic*, Translated into English by M.L. D'Ooge, With *Studies in Greek Arithmetic* by F.E. Robbins and L.Ch. Karpinski, Macmillan, New York 1926, p. 184 and Nicomaque de Gérase, *Introduction Arithmétique*, Introduction, traduction, notes et index par J. Bertier, Vrin, Paris 1978 (Histoire des doctrines de l'Antiquité classique, 2), pp. 55-6; G. Johnson, *The Arithmetical Philosophy of Nicomachus of Gerasa*, New Era Print, Lancaster PA 1916 (Reprints Univ. of Michigan Library), p. 5.

[83] See below, p. 101-2 as well as Endress, "The Circle of al-Kindī" (quoted above, n. 31), p. 55. For the influence of the first sections of Nicomachus, *Introduction to Arithmetic* on al-Kindī's treatment of mathematics in *On the Quantity of Aristotle's Books*, see also Adamson, *Al-Kindī* (quoted above, n. 32), pp. 33-5; 173.

[84] See above, n. 79.

> The human science (*al-ʿilm al-insānī*) we have defined is below the divine science (*dūn al-ʿilm al-ilāhī*) and there is no way to comprehend it together with the true and stable things, without mathematics. [...] because there are two arts that investigate quantity: one of them is arithmetic (*ṣināʿat al-ʿadad*). It investigates the quantity set apart, I mean the computational quantity (*kammiyyat al-ḥisāb*), adding [numbers] to one another or subtracting them from one another. It also happens that they are multiplied by one another (*taḍʿīf baʿḍihā bi-baʿḍ*) or divided by one another. The other science is harmonics (*ʿilm al-taʾlīf*), that is finding the proportion (*ratio*) of numerical [quantities] to one another and their combination, and knowing the consonant and the dissonant ones. These investigated things are quantities in relation to one another.
> Two arts investigate quality: one of them is the science of the quality at rest (*ʿilm al-kayfiyya al-ṯābita*), that is the science of surveying (*ʿilm al-misāḥa*) which is called geometry. The other is the science of the quality in motion (*ʿilm al-kayfiyya al-mutaḥarrika*), which is the science of the configuration of the universe (*ʿilm hayʾat al-kull*), in terms of figure and motion, including the periods of motion of each one of the bodies of the universe that are not affected by generation and corruption – until their Creator (*mubdiʿuhā*) wipes them out, if He wills, in one stroke (*dafʿatan*), in the same manner He had created them –, and what happens to [the universe]. This science is called astronomy.
> The first of these [sciences] in terms of rank as well as in teleological process, that is also worthy of being given precedence, is [the science] of arithmetic. Indeed, if there were no number there would be no numbered things and no combination of numbers (harmonics). And from among the numbered [things], there would be no lines or surfaces or bodies or periods or movements. If there were no number there would be no [science] of surveying (*ʿilm al-misāḥa*) (i.e. geometry) and no astronomy (*ʿilm al-tanǧīm*).[85] The second is geometry whose proof is great; the third is astronomy which is composed (*murakkab*) of arithmetic and geometry and the fourth is harmonic which is composed of arithmetic, geometry and astronomy.[86]

A few remarks are in order. Again, this second enumeration ends with the same claim that arithmetic should be learnt first, reproducing some of Nicomachus's argument in this regard, namely that arithmetic is the one "which naturally exists before all these [four methods]":

> Inasmuch as it abolishes other sciences with itself but is not abolished together with them [...]. If geometry exists, arithmetic must also be implied, for it is with the help of this latter that we can speak of triangle, *quadri*lateral, *octa*hedron, *icosah*edron, *double eightf*old, or *one and one-half* times or anything else of the sort which is used as a term by geometry, for it is impossible to conceive of such things without the numbers that are implied with each one.[87]

The same argument holds for harmonic, because:

> the musical harmonies, dia*tessaron*, dia*pente* and dia*pason* are named for numbers and more evidently still astronomy attains through arithmetic the investigations that pertain to it, not alone because it is

[85] *ʿIlm al-tanǧīm* is literally astrology that is 'applied astronomy', the same way *ʿilm al-misāḥa* is strictly speaking the 'science of surveying'. And just as al-Kindī has made it clear that by *ʿilm al-misāḥa* he means geometry, it is obvious that in this context he also means by *ʿilm al-tanǧīm* astronomy, and that he is using a loose terminology. The same denomination occurs in *On the String Instruments*, in *Muʾallafāt al-Kindī al-Mūsiqiyya*, p. 70.20 Yūsuf, see above, p. 94.

[86] *Fī Kammiyat kutub Arisṭūṭālīs*, in *Rasāʾil al-Kindī al-falsafiyya*, I, pp. 376.12-377.21 Abū Rīda.

[87] Nicomachus of Gerasa, *Introduction to Arithmetic*, Bk I 4, tr. D'Ooge, pp. 187-8, my emphasis, inspired by Janine Bertier's translation; see Nicomaque de Gérase, *Introduction Arithmétique*, tr. Bertier, p. 58 §4.

later than geometry in origin – for motion naturally comes after rest – nor because the motions of the stars have a perfectly melodious harmony but also because risings, settings, progressions, retrogressions (i.e. forward and retrograde motions) increases and all sorts of phases are governed by numerical cycles and quantities.[88]

More strikingly, al-Kindī divides the four mathematical sciences into two groups according to *two* principles, namely quantity and quality, rather than *one* unifying principle that is quantity, as in the *Introduction to Arithmetic*. He further distinguishes between a "moving quality" and a "quality at rest" according to which geometry is qualified as the science of "the quality at rest" whereas astronomy is the science of the "moving quality". This division does not correspond to the classification reproduced by Nicomachus of Gerasa, that is grounded on the principle of determined quantity and that characterizes dimension (πηλίκος) and not quality as the object of study of geometry and astronomy. The statement is surprising, to say the least, and has caught the attention of scholars. The first two editors of the *Epistle* suggested an error in the source that al-Kindī might have used and which should have been at the origin of the shift from dimension to quality: a copyist / translator may have read ποιόν (quality) instead of πηλίκος (dimension).[89]

Interestingly enough the same 'error' occurs in the Hebrew version of the *Introduction to Arithmetic* by Qalonymos ben Qalonymos (1317). Compared with the Greek original, the Hebrew version looks like a paraphrase rather than a translation, including lengthy interpolations lacking in the Greek text. Actually, the Arabic version which is at the origin of Qalonymos' translation is the so-far-lost Arabic version by Ḥabīb ibn Bahrīz, who translated it from Syriac into Arabic for Ṭāhir ibn al-Ḥusayn Ḏū al-Yamīnayn (d. 822), a famous general under al-Maʾmūn (d. 833). This version is said to have been revised under al-Kindī's supervision[90] and the revisor states, in the prologue, that he has not only "corrected" the translation of Ḥabīb ibn Bahrīz, but that he has also abbreviated it "in a concise discourse, without repetitions or lengthy [parts]". The "obscure" terms have been explained in order to make the text more accessible to the reader without modifying the ideas.[91]

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[88] Nicomachus of Gerasa, *Introduction to Arithmetic*, Bk I 4, tr. D'Ooge, p. 189. It is worth noting that in *On the Great Art*, at the very beginning of his prologue, al-Kindī underscores that the study of astronomy should be preceded by the study of arithmetic and geometry that are constitutive of astronomy (*qiwām hāḏihi al-ṣināʿa minhumā*) (see al-Kindī, *Fī l-Ṣināʿa l-ʿuẓmā*, p. 119.1 Aḥmad and above, p. 97).

[89] See Guidi - Walzer, "Uno scritto introduttivo allo studio di Aristotele", pp. 387-8, who consider that the division of mathematics according to the criteria of ποσόν (quantity) and πηλίκον (dimension) was common in the "late mathematicians from Nicomachus to Proclus", but still suggest that al-Kindī must have followed the model of an isagogical treatise rather than a handbook of mathematics. The two scholars conclude that "the author-model of al-Kindī, who shares such disposition for mathematics, seems therefore to be a new offshoot of the Alexandrian school, being closer to the Athenian milieu of the 5th and the 6th c. dominated by the great figure of Proclus".

[90] Or rather al-Kindī's "improved recension" was communicated orally to his students and eventually one of them (the revisor?) put it in writing. For the many layers underneath the Hebrew version, see G. Freudenthal - M. Zonta, "Remnants of Ḥabīb ibn Bahrīz's Arabic Translation of Nicomachus of Gerasa's *Introduction to Arithmetic*", in T. Langermann - J. Stern (eds.), *Adaptations and Innovations: Studies on the Interaction between Jewish and Islamic Thought and Literature from the Early Middle Ages to the Late Twentieth Century, dedicated to Professor Joel L. Kraemer*, Peeters, Paris - Dudley 2007 (Collection de la Revue des Études Juives, 44), pp. 67-82.

[91] For further information and for a critical edition together with an annotated translation of the prologue and the introduction of the Hebrew translation, accompanied by an extensive commentary from which the present paragraph draws, see Freudenthal - Lévy, "De Gérase à Bagdad" (quoted above, n. 79), pp. 479-543, esp. pp. 482-3, 491-2 (pp. 513-43 for the critical edition and French translation); see p. 516 and compare with the prologue of *Fī l-Ṣināʿa l-ʿuẓma* (p. 119 Aḥmad) where al-Kindī criticizes the often 'obscure' style of the translators.

Finally, "concerning each point, [he] will not fall short of mentioning the opinion of [his] master Abū Yūsuf". We are thus dealing with a paraphrase rather than a proper translation. Among the many interpolations with which it is fraught, there are glosses attributed to al-Kindī and explicitly introduced by "Abū Yūsuf said". This being said, the following passage is a straightforward translation of the original text of Nicomachus and reads as follows:

> Il a donc été mis en évidence que la science de la quantité est l'objet des investigations de deux arts: l'arithmétique qui étudie la quantité séparée et la musique qui étudie la quantité en relation. S'agissant de la qualité (*ʾekhut*) des choses naturelles, elle se divise en deux classes: la qualité en mouvement et la qualité en repos. [La qualité] en mouvement est objet d'étude pour l'art de l'astronomie; et [la qualité] au repos est objet d'étude pour l'art de la géométrie.[92]

In a long footnote, the editors/translators of the text have drawn attention to the unusual notions of "quality in movement" and "quality at rest" suggesting also an error that might have affected, this time, the Arabic translation, which would have rendered the Greek πηλίκος ('dimension') by *kammiyya*, which would have been later on corrupted into *kayfiyya*.[93] This is not the place to discuss further the validity of such assumptions. A quick comparison with the original Greek version of Nicomachus's *Introduction* (see below, p. 104-5), as well as with Ṯābit ibn Qurra's (d. 901) translation into Arabic, which renders the Greek πηλίκος by *al-misāḥa*, shows to be conclusive.[94] It is now clear enough that at some point, the version of Nicomachus of Gerasa's *Introduction to Arithmetic* that al-Kindī was perusing has been altered. Moreover, the occurrence of the same 'error' in the Hebrew translation of the revised Arabic version of Ḥabīb ibn Bahrīz as well as in al-Kindī's *Epistle On the Quantity of Aristotle's Books*, is a strong enough indication that al-Kindī had at hand Nicomachus's text from which he was drawing when writing the *Epistle On the Quantity of Aristotle's Books*.

Indeed the *Introduction to Arithmetic* looms large in al-Kindī's *Epistle On the Quantity of Aristotle's Books.* Moreover, the quadripartition of mathematics he found in the 'altered' translation he had at hand echoes the definition of mathematics provided by Ptolemy as dealing with quantity and quality since it investigates "forms and motion from place to place" but also "shapes, number, size and place" (see above, p. 87). Along the same lines, it is worth noting that in *Harmonics*, III 3, Ptolemy has paired up, without further explanation, arithmetic and geometry as "indisputable instruments" both astronomy and harmonics use "for the quantity and quality of the first movements".[95] At any rate, such a quadripartition allowed al-Kindī to connect the division of the four mathematical sciences with the doctrine of the categories through the attributes of quantity and quality that he had singled out, at the very beginning of the *Epistle* and right after the first enumeration of the four mathematical sciences, as the first two attributes of the substance. Being the science of quantity and quality, mathematics becomes the royal road to philosophy, yielding knowledge of the "true

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[92] Freudenthal - Lévy, "De Gérase à Bagdad" (quoted above, n. 79), pp. 530-3. For the sake of precision, I decided to quote the excellent French translation of the authors of the article rather than producing a second hand English translation.

[93] *Ibid*, p. 532 n. 99.

[94] See *Ṯābit Ibn Qurra's Arabische Übersetzung der Ἀριθμητικὴ Εἰσαγωγή des Nikomachos von Gerasa zum Ersten Mal herausgegeben*, ed. W. Kutsch, Impr. Catholique, Beyrouth 1958, p. 14.9-11: *ʿilmān āḫarān yuʿraf bihimā umūr al-misāḥa: aḥaduhumā yuʿraf bihi amr al-šayʾ allaḏī lā yataḥarrak wa-huwa ʿilm al-handasa wa-l-āḫar yuʿraf bihi amr al-šayʾ al-mutaḥarrik allaḏī yadūr wa-huwa ʿilm al-kura*.

[95] See Solomon, *Ptolemy Harmonics* (quoted above, n. 70), p. 142 and n. 85; see also above, p. 97.

nature of things",[96] as al-Kindī shows in an argument that remains very close to the first sections of Nicomachus's *Introduction*, as we will see below.

*II.ii Mathematics as Knowledge of Reality through Quantity and Quality*

The argument unfolds in a long development that sheds also some light on al-Kindī's epistemology.

> Since the first object of knowledge (*al-maʿlūma l-ūlā*) that is comprehended by each philosophical science is the substance along with quantity and quality, and since the first substance – I mean the sensible one – is also comprehended through the knowledge of its first attributes – for the sense does not apprehend (*yubāšir*) the first substance directly, rather it apprehends it through the intermediary of quantity and quality –, him who lacks the knowledge of quantity and quality lacks on top of that the knowledge of the substance.
> The stable, true and perfect knowledge (*al-ʿilm al-ṯābit al-ḥaqqī al-tāmm*) of the science of philosophy is the knowledge of the substance. The secondary substances are the ones whose knowledge does not disappear (*lā zawāl li-ʿilmihā*), because of the stability of the object of knowledge and its remoteness from change and flux (*al-tabaddul wa-l-sayalān*). It can be reached only through the knowledge of the first substance.
> As for the sensible knowledge, it is the knowledge of the first substance, because of the uninterrupted flux of its object of knowledge – which ends (*nafada*) only when [the object of knowledge] ends, that is when its substance is totally destroyed. Or because of the multiplicity of the sensible substance regarding numerical multiplicity (*fī kaṯrat al-ʿadad*) ([i.e.] even if every numbered thing is [in itself] limited, it is [nevertheless] possible to increase each numbered thing by its double) which can be potentially increased infinitely – [i.e.] not in the [actual] number of individuals or in the [actual] number of multiplications of the increase. And what is infinite is not comprehended by a science. Hence him who lacks the knowledge of quantity and quality lacks the knowledge of the first and secondary substances, as we have already mentioned.[97]

The argument presupposes that one of the fundamental properties of the first sensible substances, besides being transient and in constant flux, is to be multiple. Multiplicity, if not limited, can be potentially[98] increased infinitely, starting from a determined unity, and there is no science of what is infinite. However, science can deal with a determined multiplicity, meaning with "how many" (*kamm*) and hence with quantity. According to Aristotle, "a *quantum* is multitude if it can be numbered and a magnitude if it can be measured".[99] Thus knowledge of quantity, and hence

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[96] See *Fī Kammiyat kutub Arisṭūṭālīs*, in *Rasāʾil al-Kindī al-falsafiyya*, I, p. 364.6 Abū Rīda.

[97] *Fī Kammiyat kutub Arisṭūṭālīs*, in *Rasāʾil al-Kindī al-falsafiyya*, I, p. 372.2-14 Abū Rīda. For this passage, see also Adamson, *Al-Kindī* (quoted above, n. 32), pp. 129-30 and 230, n. 40, who also noted that Nicomachus of Gerasa's *Introduction to Arithmetic* I 1 is a source for al-Kindī's theory of flux and identification of secondary substances as the objects of knowledge.

[98] Note al-Kindī's consistency as he rejects the existence of an actual infinite, but seems to accept the existence of a potential infinite, a position he held in several of his treatises. Compare this position especially with his *Epistle On the Quiddity of What cannot be Infinite and What is Called Infinite*, in Rashed - Jolivet, *Œuvres Philosophiques* (quoted above, n. 3), vol. II, p. 155.13-20 (a passage that comes very close to the one at stake); see also *On First Philosophy*, as well as his *Epistle On the Unicity of God and the Finiteness of the Universe*, *ibid.*, p. 31.14-20; p. 141.9-15. Incidentally, Aristotle defines plurality in *Metaph.* V 13, 1020 a 7 as "that which is divisible potentially into non-continuous parts".

[99] See *Metaph.* V 13, 1020 a 9, quoted by F.E. Robbins, "Nicomachus's Philosophy of Number", in Nicomachus of Gerasa, *Introduction to Arithmetic*, tr. D'Ooge, p. 112.

mathematics, will be the only possible way to reach the knowledge of the first substance and by the same token of the secondary substances since these cannot be reached otherwise. The statement is decidedly Aristotelian, and reflects the doctrine of the *Categories* that places the emphasis on the primary substance "everything else being either said of the primary substances as subjects or in them as subjects" (*Cat.* 5, 2 a 35). Thus, if there were no primary substances, there would be no "secondary" substances, i.e. no species and genera, either. The statement is not only true ontologically but also epistemologically. If it is true that there is no science of the particular, it is equally true that we can reach the universal through perception.[100] Hence, even if the secondary substances are constitutive of the only permanent and stable knowledge which is not exposed to "change and uninterrupted flux", the hegemony of the first substance is such that the secondary substances can only be apprehended through the primary one. However, the sheer multiplicity of the sensible can only be grasped if limited and thus numbered in order to become the object of mathematics. This is why al-Kindī states that "the sense does not apprehend directly" the first substance but "through the intermediary of quantity and quality". Multiplicity *per se* is indefinite and infinite[101] and as such cannot be the object of any knowledge or science.

In the background looms the first chapters of Nicomachus of Gerasa's *Introduction to Arithmetic* that opens with the famous definition of philosophy ("love of wisdom") attributed to Pythagoras, who is said to have been the first to have restricted the word wisdom (σοφία) to "the knowledge or science of the truth in the things that are, conceiving [...] 'the things that are' to be those which continue uniformly and the same in the universe and never depart even briefly from their existence".[102] Accordingly, Nicomachus divides all things in the universe into those that "are called being in the proper sense", that is the unchanging immaterial and intelligible realities, and those which are only "called beings" by homonymy, namely the ever changing sensible things whose existence derive from the true beings. All these things, both the eternal immaterial as well as the ever changing material are either continuous – and those are called 'magnitudes' (μεγήθη) – or discrete (juxtaposed and 'heap-like') that are labeled "multiplicities" (πλήθη). "Wisdom, then, must be considered to be the knowledge of these two forms". However, multiplicity and magnitude are by their own nature infinite, and hence cannot as such be the object of science, since there is science only of what is determined and never of what is limitless.[103] "A science, however, would arise to deal with something separated from each of them, with quantity (τὸ ποσόν), set off from multitude and dimension (τὸ πηλίκον) set off from magnitude." Therefore:

> Since of quantity one kind is viewed by itself, having no relation to anything else, as 'even', 'odd', 'perfect' and the like, and the other is relative to something else and is conceived of together with its relationship to another thing, like 'double', 'greater', 'smaller', 'half', 'one and one-half times', 'one and one-third times' and so forth, it is clear that two scientific methods will lay hold of and deal with the whole investigations of quantity; arithmetic absolute quantity, and music relative quantity. And once

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[100] See above, p. 93.

[101] See Robbins, "Nicomachus's Philosophy of Number" (quoted above, n. 98), p. 112.

[102] For the quotations in this paragraph, see Nicomachus of Gerasa, *Introduction to Arithmetic*, I 1-3 tr. D'Ooge, pp. 181-4 slightly modified. See also I. Hadot, *Arts libéraux et philosophie dans la pensée antique. Contribution à l'histoire de l'éducation et de la culture dans l'Antiquité*, Vrin, Paris 2005² (Textes et traditions), pp. 64-9 and Robbins, "Nicomachus's Philosophy of Number" (quoted above, n. 98), pp. 111-13, on both of which this paragraph draws.

[103] See Aristotle, *Metaph.* V 13, 1020 a 14: "limited plurality is number, limited length is a line, breadth a surface, depth a solid".

more, inasmuch as part of 'dimension' is in a state of rest and stability, and another part in motion and revolution, two other sciences in the same way will accurately treat of 'dimension', geometry the part that abides and is at rest, astronomy that which moves and revolves. Without the aid of these, then it is not possible to deal accurately with the forms of being nor to discover *the things that are*, knowledge of which is wisdom, and evidently not even to philosophize properly.[104]

It is now clear that regardless of the 'altered' translation he had at hand, that replaces dimension (πηλίκον) by quality (ποιόν), al-Kindī drew from the *Introduction to Arithmetic* the epistemological framework and the argument that allowed him to erect mathematics as the method and the science required to reach a knowledge of the "truthful and stable things" (*al-ašyāʾ al-ḥaqqiyya al-ṯābita*).[105] It remains to show what are the "the truthful and stable things," or to put it differently, to what extent do they overlap with the beings, that are the object of philosophy according to the Phythagorean definition quoted by Nicomachus.

*II.iii The Secondary Substances and the Eternal Realities*

The version of Nicomachus's *Introduction to Arithmetic* that is said to have been revised under al-Kindī's supervision reproduces, next to the Pythagorean definition of philosophy as "the true (veridical) knowledge of the [eternal] things" a gloss identifying the "eternal things" as "species and genera".[106] The gloss might be attributed either to al-Kindī himself (even though it is not introduced by "Abū Yūsuf said"), or to the revisor, who is said to have been al-Kindī's student, recounting his master's comments.[107] In either case we are dealing with al-Kindī's teaching.

As we have seen above, the definition of philosophy attributed by Nicomachus to Phythagoras as "the knowledge or science of the truth in the things that are" further specifies that the "things that are" are "those which continue uniformly and the same in the universe and never depart even briefly from their existence". Accordingly, the glossator has understood the "things that are" as the eternal realities that he further identifies with species and genera, whereas in the *Introduction* the "eternal things" are described as real transcendent beings. Species and genera belong rather to the vocabulary of the *Categories* and the famous distinction between the primary substance and "the species in which the things primarily called substances are, [that] are called *secondary substances*, as also are the genera of these species" (*Cat*. 5, 2 a 15). Does the interpolation aim at identifying the "eternal things" with the secondary substances of the *Categories*? That would entail applying on the Platonic ontological division drawn by Nicomachus between the transcendent, eternal and immaterial beings, on the one hand, and "everything else that exists under the same name and is so called [which] is said to be 'this particular thing'[108] and exists"

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[104] See Nicomachus of Gerasa, *Introduction to Arithmetic*, I 3, tr. D'Ooge, p. 184 slightly modified (my emphasis).

[105] *Fī Kammiyat kutub Arisṭūṭālīs*, in *Rasāʾil al-Kindī al-falsafiyya*, I, p. 376.13 Abū Rīda.

[106] See Freudenthal - Lévy, "De Gérase à Bagdad" (quoted above, n. 79) p. 518: "<u>Pythagore, quant à lui</u>, réservant ce nom à son [véritable] objet et au principe dont il est dérivé, <u>a appelé science en un sens spécifique</u> – à l'exclusion des sciences qui en découlent – <u>la connaissance véridique des choses</u> éternelles, *c'est-à-dire les espèces et les genres*)", my emphasis (italics). The underlined phrases are those signaled by the editors as corresponding to the original text, whereas the rest of the passage is considered to be an interpolation. It is worth noting that the editors did not underline "eternal", which should thus be considered as part of the gloss.

[107] See the prologue of the text in Freudenthal - Lévy, "De Gérase à Bagdad" (quoted above, n. 79), pp. 514-16.

[108] Τόδε τι is the technical expression used by Aristotle to designate the particular thing or primary substance, as pointed by D'Ooge (see Nicomachus of Gerasa, *Introduction to Arithmetic*, tr. D'Ooge, p. 181 and n. 3).

on the other, an Aristotelian framework that is after all not completely alien to the *Introduction to Arithmetic*.[109]

The passage mentioned above (p. 105 and n. 106) is followed by a long interpolation, introduced by "Abū Yūsuf said" and listing six different definitions of philosophy. As has been already noticed by the editors and translators of the text, it is an excerpt from al-Kindī's *Epistle On the Definitions of Things and their Descriptions*. The sixth and last one is clearly introduced by al-Kindī as his own, and defines philosophy as: "the knowledge *of the universal and eternal objects*, of their beings, their essences, their causes and their reasons".[110]

A further interpolation, a few lines after, glossing on the definition of science as the "true knowledge of eternal things", describes the eternal and changeless things as "the species and the first natural genera by participation in which the individuals are worth being called 'existent' since their species bestow on them names and definitions".[111] The semantic field is clearly that of the participation of the particular changing things in the eternal and changeless ones that bestow on them not only names and definitions, but also existence. Nevertheless, the "eternal things" are described again as species and genera in a statement that is reminiscent of the description of the secondary substances, in *Cat*. 5, 2 a 20, that are 'said of' a subject, namely "both their name and their definition are necessarily predicated of the subject".

Such an interplay between the Platonico-Pythagorean theory and the Aristotelian doctrine of the *Categories* finds an ultimate echo in the *Epistle On the Quantity of Aristotle's Books* where, as we have seen above, al-Kindī identifies the "hidden secondary substances" with the "the true and stable things" whose knowledge requires nevertheless the prior knowledge of the first sensible substance.[112] Now it becomes easier to understand why in the first enumeration of Aristotle's books,

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[109] The proximity between the terminology of the *Categories* and the vocabulary used by Nicomachus in his *Introduction* has been already noticed in scholarship. On this issue see particularly D. O'Meara, *Pythagoras Revived: Mathematics and Philosophy in Late Antiquity*, Oxford U.P., Oxford 1989, pp. 16-17.

[110] See Freudenthal - Lévy, "De Gérase à Bagdad" (quoted above, n. 79), p. 522: "Quant à nous [al-Kindī] nous définissons la philosophie en disant que la philosophie est connaissance *des objets universaux et éternels*; de leurs êtres, leurs essences, leurs causes et leurs 'pourquoi', et ce, selon ce que peut atteindre l'homme. Fin des paroles d'Abū Yūsuf" (my emphasis). For the full enumeration, see pp. 518-22. Compare with the same definition in *On the Definitions of Things and their Descriptions*, that defines philosophy as "the knowledge of eternal universal things (*ʿilm al-ašyāʾ al-abadiyya al-kulliyya*): of their existences, of their essences of their causes to the extent of man's capacity (*bi-qadar ṭāqat al-insān*)" (*Risālat Yaʿqūb b. Isḥāq al-Kindī Fī Ḥudūd l-ašyāʾ wa-rusūmihā*, in *Rasāʾil al-Kindī al-falsafiyya*, I, p. 173 Abū Rīda and al-Kindī, *Cinq Épîtres* [quoted above, n. 7], p. 23). As noted by Freudenthal and Lévy (p. 522 n. 87), this definition has been compared with the one with which al-Kindī opens his treatise *On First Philosophy*, describing "the art of philosophy" as "the science of the things in their true natures to the extent of man's capacity"; see al-Kindī, *Cinq Épîtres* (quoted above, n. 7), p. 60, § 70f and *Kitāb al-Kindī ilā al-Muʿtaṣim bi-llāh Fī l-falsafa l-ūlā*, in *Rasāʾil al-Kindī al-falsafiyya*, p. 97 Abū Rīda, and Rashed - Jolivet, *Œuvres Philosophiques* (quoted above, n. 3), vol. II, p. 9. See also above, n. 78.

[111] See Freudenthal - Lévy, "De Gérase à Bagdad" (quoted above, n. 79) p. 522: "L'auteur de ce livre [Nicomaque] dit: <u>De plus, il [Pythagore] a défini la science en disant que la science est la connaissance véridique des choses éternelles. Il a également défini la connaissance en disant que la connaissance est l'appréhension du *telos* des choses qui sont objet de la connaissance; celles qui sont éternelles, dont l'existence ne se modifie pas, dont la quiddité ne change pas, et dont la propriété ne subit pas de mutation</u>. *Ce sont les espèces et les premiers genres naturels*, qui – lorsque les individus y participent et sont définis par eux – méritent le nom d'existant, puisque leurs espèces leur confèrent noms et définitions" (my emphasis).

[112] See *Fī Kammiyat kutub Arisṭūṭālīs*, in *Rasāʾil al-Kindī al-falsafiyya*, I, p. 372.7-8 Abū Rīda quoted above, p. 103.

at the beginning of his *Epistle*,[113] while all the other books were listed by title only with no or very little comment, al-Kindī singled out the *Categories* by a long development on the substance and its attributes, emphasizing the distinction between the two kinds of predicables, namely those which are 'said of' the subject synonymously and hence confer upon it their name and definition, and those which are predicated homonymously (*bi-ištibāh al-ism*), that is the accidents which are 'in' the subjects and hence neither the name nor the definition is predicated of the subject (*lam yuʿṭihi ḥaddahu wa-lā ismahu*). Actually al-Kindī was paving the way for the discussion that will follow concerning the necessity to precede the study of philosophy, here encompassed in Aristotle's books and understood as the knowledge of the substance, by the knowledge of mathematics in its four branches. The secondary substances are predicated of the primary sensible ones, as al-Kindī specifies by the end of the *Epistle* when listing again Aristotle's books according to their "intentions" (*aġrāḍ kutub Arisṭūṭālīs*), and hence they give their name and definition to the first substances, but the former can only be apprehended through the latter, the knowledge of which requires mathematics, as has been already shown.

It becomes also easier to understand why in this first development the only category besides the substance that has been singled out by a brief description is the category of quality, represented by two of its subdivisions, that is "color" and "figure", as has been already noted by Jolivet, who underscores that the choice of the latter is not insignificant in that context.[114] Indeed a few lines below al-Kindī specifies that the "first object of knowledge" for he who wants to investigate the books of Aristotle that have been already listed, is the substance, through the knowledge of its first two attributes, namely quantity and quality.[115]

### *III. Coming Full Circle*

By promoting quality and quantity as the leading path towards the knowledge of the first and secondary substances, al-Kindī elevates the science of quantity and quality, namely mathematics, as the best guide to reality.[116] Indeed such statements echo the description of mathematics (quoted above, p. 91-93) that al-Kindī provides in *On the Great Art*, as related to the form embedded in matter, meaning surfaces and limits and also by the "quality of the form" like figures and shapes, triangles, squares and suchlike as well as by any determined quantity like size, number, time, and place. Such a definition reflects, as we have already seen, Ptolemy's own description of mathematics in the preface of the *Almagest*. Consequently mathematical objects are considered as "an attribute of all existing things without exception, both mortal and immortal" and hence mathematics will be the only theoretical science that yields knowledge of all reality (which incidentally includes first and secondary substances). By comparison, physics and theology should be described as conjecture rather than a science that can yield knowledge, "theology because of its completely invisible and ungraspable nature and physics because of the unstable and unclear nature of matter".

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[113] *Ibid.*, pp. 365-6.

[114] See Jolivet, "L'*Epître Sur la Quantité*" (quoted above, n. 77), p. 671.

[115] See *Fī Kammiyat kutub Arisṭūṭālīs*, in *Rasā'il al-Kindī al-falsafiyya*, ed. Abū Rīda, I, p. 370.9-13: "It is evident that the first object of knowledge, for him who investigates the things we have already mentioned [i.e. Aristotle's books he has just listed], is the knowledge of the substance and the attributes of the substance. The first simple attributes of the substance (*maḥmūlāt al-ǧawhar al-ūlā l-mufrada*) are two, namely quantity and quality, because any of the attributes that attach to the substance differ either by equal and unequal (*miṯl wa-lā miṯl*) which is the specific property of quantity or by similar and dissimilar *(šabīh wa-lā šabīh*) which is the specific property of quality".

[116] See above, n. 26.

Ptolemy goes even further by explaining how mathematics can contribute significantly to the other two sciences, particularly to theology in that "it alone can make proper inferences about the unmovable and separate actuality on the basis of its nearness to the properties of perceptible beings that are, on one hand causes of motion and moved, and on the other, eternal and impassive". [117] He is of course referring to the aetherial astronomical objects that are on the one hand perceptible but on the other eternal and unchanging "inasmuch as the only change they experience is motion from place to place".[118] When Ptolemy claims that theology is conjectural he grounds his argument on the "non-evident and ungraspable" character of the Prime Mover and accordingly gives precedence to mathematics as the only kind of theoretical philosophy "that can provide sure and incontrovertible knowledge". Then, the question arises: how far can al-Kindī follow Ptolemy in his epistemological 'demotion' of theology as inferior to mathematics in terms of yielding knowledge, knowing that he had already qualified elsewhere "the science of the first Cause" as "the noblest part of philosophy being first in nobility, first in genus and *first from the point of view of what is scientifically the most certain*"?[119]

Interestingly enough, in *On the Great Art*, the description of the object of theology as being the motionless, imperceptible and ungraspable separate Prime Mover, "that cannot be grasped by any science that would comprehend it",[120] fits quite well with al-Kindī's account of the absolute transcendence of the True One and First Cause (regardless of the conceptual difference between the First Cause and the Prime Mover) above and beyond any description in *On First Philosophy*. In that respect al-Kindī was being quite consistent. However, according to the epistemological criteria applied in the text he is paraphrasing, theology falls short of the certainty of mathematics, even though he does not go as far as qualifying it as conjectural, but emphasizes the "ungraspable" character of its object that cannot be "comprehended by a science".

It is actually the *Epistle On the Quantity of Aristotle's Books* that restores the epistemological primacy of theology while preserving the precedence of mathematics. By restricting the "divine science" (*al-ʿilm al-ilāhī*) to the prophets and describing it as an immediate "illumination" directly inspired by God as opposed to the laborious "human sciences" (*al-ʿulūm al-insāniyya*) that need to resort to "the devices of mathematics and logic", al-Kindī overrides the certainty produced by the indisputable procedures of mathematics with the immediate access of the prophets to the truth. At the same time, theology and mathematics are brought into line as sharing the same content.

> We cannot hope for him [who lacks the knowledge of quantity and quality and hence lacks the knowledge of the first and secondary substances] to ever know any of the *human sciences* (*al-ʿulūm al-insāniyya*), which are the object of mankind's (*al-bašar*) pursuit and effort and of their contrived devices (*ḥiyaluhum al-maqṣūda*), and hence are ranked below the divine science which is not the object of any pursuit, nor effort, nor human devices or time, such as the science/knowledge of the apostles (*al-rusul*), may God's prayers be upon them, with which God, great and most highly Exalted, endowed them specifically. [The science of the prophets] is attained without being sought

[117] Ptolemy, *Almagest*, I 1, p. 7.5-9 Heiberg; translated in Bowen, "The Demarcation of Physical Theory and Astronomy" (quoted above, n. 21), p. 351.

[118] See Feke, "Ptolemy in Philosophical Context" (quoted above, n. 20), p. 60.

[119] See *On First Philosophy*, in *Rasāʾil al-Kindī al-falsafiyya*, I, p. 101.16-17 Abū Rīda, and Rashed - Jolivet, *Œuvres Philosophiques* (quoted above, n. 3), vol II, p. 11.14-15: *ūlā bi-l-šarafi wa-ūlā bi-l-ǧinsi wa-ūlā bi-l-tartībi min ǧiḥati al-šayʾi al-ayqani ʿilmiyyatan*.

[120] See al-Kindī, *Fī l-Ṣināʿa al-ʿuẓmā*, p. 127.7 Aḥmad.

> of, with no effort or research, *with no devices of mathematics and logic* and with no time, rather with His will, Mighty and Exalted He be, by purifying their souls and illuminating them for the truth, with His support, His guidance, His inspiration and His messages. This science is proper to the Apostles, God's prayers be upon them, to the exclusion of human beings [...]. There is no path for any human being other than the Apostles, to the *momentous knowledge of the science of the secondary hidden substances and to the science of the first sensible substances* and what happens accidentally in them, neither by being sought *nor by the devices of logic and mathematics* that we have mentioned and with time [...].[121]

Human science is contrasted with divine science in terms of method, not in terms of content[122] since both of them share in the same object, namely the knowledge of "the secondary hidden substances" as well as of "the first sensible substances". They differ in that divine science is immediate, and does not require any effort or device; rather, the soul is "illuminated" by the truth as a result of the will of God. By contrast, human science requires the devices of logic and mathematics in order to reach the same knowledge. It is worth noting that mathematics and logic are twice mentioned in pair: they are thus considered as sharing in the same nature and fulfilling the same function. Like logic, mathematics is also considered as an *organon*, and hence a method that guides the thought of the philosopher towards the "true and stable knowledge" through an ingenious exercise of reason on the first sensible substance in order to transform it into an object of knowledge expressed in terms of quantity and quality. Therefore, while there is no doubt that human science owes its inferior status to the necessity to resort to the devices of mathematics and logic, at the same time mathematics is elevated in rank since it is the only method through which mankind's philosophy (*falsafat al-bašar*) can reach, through a long and fastidious digression, the content of the divine science that is immediately accessible to the prophets.

> As for the human science (*al-ʿilm al-insānī*) we just defined, it is inferior to the divine science, and there is no way to comprehend it together with the true and stable things (*al-ašyāʾ al-ḥaqqiyya l-ṯābita*), without mathematics, except with the capacity of apprehension through the senses alone that the irrational animal does not lack [...].[123]

Hence the necessity of preceding the study of philosophy, here encompassed in Aristotle's books, with a knowledge of mathematics in its four branches. The didactic necessity of the propaedeutic status of mathematics is on a par with its epistemological position in the middle between the pure sensible knowledge proper to "the irrational animal" and the divine science "of the secondary hidden substances" now identified with the "true and stable things".

The same tension pervades *On the Great Art*. When al-Kindī ranks mathematics in the middle between physics and theology he actually grounds his classification, in the footsteps of Ptolemy, on epistemological criteria related first to the perceptibility or imperceptibility of the object proper

[121] *Fī Kammiyat kutub Arisṭūṭālīs*, in *Rasāʾil al-Kindī al-falsafiyya*, I, pp. 372.15-373.7 Abū Rīda (my emphasis).

[122] See Adamson, *al-Kindī* (quoted above, n. 32), pp. 43-4 who also highlights the similarity between the prophetic knowledge and the philosophical one, the difference between both residing only in the immediate and effortless access of the prophets "to the same truths", and points to al-Kindī's *Epistle On the Reason Why the Upper Atmosphere Becomes Cold and That Which is Closer to the Earth Becomes Hot*, where the same idea is reiterated in almost the same terms. See *Risāla fī l-ʿilla allatī lahā yabrudu aʿlā al-ǧaww wa-yasḫunu mā qaruba min al-arḍ*, in *Rasāʾil al-Kindī al-falsafiyya*, II, pp. 92-3 Abū Rīda.

[123] *Fī Kammiyat kutub Arisṭūṭālīs*, in *Rasāʾil al-Kindī al-falsafiyya*, I, p. 376.12-14 Abū Rīda.

to each science (mathematical objects can be grasped through sense perception and apart from sense perception, by intellect) and second to the fact that the mathematical objects are a property of absolutely all things, both mortal and immortal. And it is precisely its intermediary position between sense-perception and intellect, paired with the usage of "indisputable procedures" that leads both authors to give precedence to mathematics over the other two sciences in terms of yielding knowledge. Not only there is no contradiction between the intermediary position of mathematics as a theoretical science and its epistemological primacy, but the degree of certainty it provides is precisely dependent on its position in the middle between sense perception and intellect, as it uses both (namely a geometric model consistent with observations) in its production of knowledge.

*Conclusion*

The revised version of Ḥabīb ibn Bahrīz's translation of the *Introduction to Arithmetic*, that came down to us in the Hebrew version of Qalonymos b. Qalonymos, is preceded by a prologue by the hand of the revisor in which the latter informs us that more than once he has heard his master (i.e. al-Kindī) saying that: "the philosophy of these two figures, namely Ptolemy and Nicomachus, is best expressed in the introductions to their books, for Ptolemy in the introduction to the *Almagest* and for Nicomachus in the introduction to his *Book On Arithmetic*. Indeed, the introductions of both books touch upon the noblest topics of philosophy and they occupy a high rank with respect to knowledge".[124] The fact that al-Kindī brings Ptolemy and Nicomachus together under the same banner in terms of philosophical interest – despite the fact that they belong to two different traditions – is indeed telling. Besides having both been considered as great mathematicians, they both identified themselves as philosophers and wrote about the relationship between mathematics and philosophy. The introductory sections of their major works, respectively the *Almagest* and the *Introduction to Arithmetic*, reflect their views in that regard. Al-Kindī's work bears the strong imprint of both philosophers. The influence of Ptolemy is apparent way beyond the paraphrase of the beginning of the *Almagest*. He was familiar with most of Ptolemy's works. He knew the *Tetrabiblos* and Ptolemy's *Geography* and most probably also the *Harmonics*. As for Nicomachus, al-Kindī wrote an Epistle on the *Introduction to Arithmetic* that unfortunately did not reach us.[125]

The centrality of mathematics in al-Kindī's philosophy has already been highlighted in previous scholarship, and traced back to the influence of both Ptolemy and Euclid. The mathematical epistemological approach that pervades most of al-Kindī's work and provides him with a scientific method that he applies even in his philosophical argumentation has been underlined, particularly the geometrical proof that he has found in Euclid as well as in Ptolemy's introduction to the *Almagest*. This article has attempted to show that al-Kindī has drawn from Ptolemy more than an epistemological tool. Rather, he inherited a philosophical system that gives a central position to mathematics, precisely on account of its classification as a full theoretical science in the middle "between the other two divisions of theoretical philosophy that should be called guesswork rather than knowledge". A position it owes in part to "its kind of proofs that proceed by indisputable methods, namely arithmetic and geometry". However, beyond the production of "a sure and unshakable knowledge" mathematics is also a science "desirable for itself" and the best mean to reach not only a better

[124] Freudenthal - Lévy, "De Gérase à Bagdad" (quoted above, n. 79) p. 516.

[125] *Kitāb Risālatihi fī l-Madḫal ilā l-ariṯmāṭīqī, ḫams maqālāt* (*K. al-Fihrist*, p. 256 Flügel).

understanding of the celestial movements but also a more harmonious ethical disposition.[126] Hence mathematics is endowed with a practical dimension (cultivating the right disposition of the soul that leads ultimately to the imitation of God) correlative to an ontological one, that is the relationship with the supranatural through its effects, namely the eternal and unchanging movements of the celestial spheres.

Ultimately, Ptolemy and Nicomachus constitute the framework within which al-Kindī's philosophy unfolds, deploying of course other sources of influence, at the top of which is Aristotle whom he calls "the foremost among the Greek philosophers". The Arabic Plotinus, that is the *Theology of Aristotle* and its corollary texts, as well as Proclus' *Elements of Theology*, among several others, have also had their share of influence on al-Kindī.[127] However, Ptolemy's preface to his *Almagest* combined with the introductory sections of Nicomachus' *Introduction* provided him with a model that structured all the aspects of his philosophy. This article has highlighted their influence on one of them, that is the relation between the mathematical sciences and philosophy.

[126] See A. Bernard, "The Alexandrian School. Theon of Alexandria and Hypatia", in Gerson (ed.), *The Cambridge History of Philosophy in Late Antiquity* (quoted above, n. 17), pp. 417-36 (p. 424).

[127] For a thorough enumeration and analysis of the full range of al-Kindī's sources, see Endress, "Building the Library of Arabic Philosophy" (quoted above, n. 32).